Logic as a science of patterns? *

The position of logic *vis-à-vis* the sciences has always been somewhat peculiar. Until the nineteenth century, logic was taken to belong to philosophy, which would indicate that it is not really a science. However, from its inception (in the work of Aristotle) it was considered as more of a *tool* of philosophy than as its component, and as at this time the sciences were not yet clearly differentiated from philosophy, perhaps the tool of the sciences too. Moreover, in the nineteenth and twentieth century, logic spilled over from philosophy and colonized a part of mathematics. Hence nowadays many people would say that of course logic is a science, because it is a sub-discipline of mathematics (or theoretical computer science or something of the kind).

However, we should not forget that the constitutive task of logic is to help us reason, to help us decide which arguments are correct and which not[1]. And as reasoning and arguments are things that occur in the real world, it seems that if we want to take logic as a kind of mathematics, it should be more of an *applied* than a pure mathematics. In this sense, logic seems to be akin to a natural science, capturing real world phenomena by means of picking up their structures and analyzing them by means of mathematics. And indeed, in this paper I am going to argue that despite substantial differences between logic and natural sciences, this might be a useful way to look at logic.

I am going to propose that logic may be seen as a science of patterns; however, not in the sense in which mathematics is a science of patterns, but more in the sense in which physics is. Logic, I am going to argue, can be seen as trying to get a grip on the inferential patterns *de facto* governing our argumentative practices. True, it does not pursue its patterns in *exactly* the same way as physics pursues its ones; but drawing a parallel between the two enterprises may be enlightening.

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* Work on this paper was supported by Grant No. 20-18675S of the Czech Science Foundation. The author is grateful to Vladimír Svoboda and two reviewers of this journal for valuable critical comments.

[1] Charles Sanders Peirce ("Logic", in J. M. Baldwin, ed., *Dictionary of Philosophy and Psychology*, vol. 2, New York: MacMillan, 1902, pp. 20–23 at p. 20–21) writes: "Nearly a hundred definitions of [logic] have been given. It will, however, generally be conceded that its central problem is the classification of arguments, so that all of those that are bad are thrown into one division, and those which are good into another." See Georg Brun, "Formalization and the Objects of Logic", *Erkenntnis* LXIX, 1 (2008): 1–30.

I. PATTERNS LOGIC IS INTERESTED IN

More than twenty years ago, Michael Resnik called his book *Mathematic as a science of patterns*[2], to give voice to a strengthening conviction held by many philosophers of mathematics that mathematics is not about any particular objects, but about structures[3]. In a recent book Angela Potochnik[4] claims that there is a sense in which natural science is also about patterns, namely that its primary aim is seeking *causal* patterns.

Needless to say, these two senses of 'being about patterns' are very distinct. Mathematics is about patterns in the abstract sense, it studies (analyzes, classifies, anatomizes ...) patterns *qua* patterns. It can be about any structure whatsoever (though, of course, it tends to focus on certain "interesting" structures). Natural science, on the other hand, is about the very specific kind of patterns that it finds when it inspects concrete reality from the viewpoint of its causal order.

Logic is also sometimes considered as a science of patterns in the abstract sense, as studying abstract structures as mathematics does, only different ones. (For example, the movement of "Universal Logic" was based on this very supposition[5].) Here, however I want to suggest that logic can be seen as a science of patterns more in the sense of natural science like physics than in the sense of mathematics.

Why should logic not be seen as a science of patterns in the sense of mathematics? In my opinion, it is because there is no such thing as an "inherently logical" (in contrast to "mathematical") structure. There are just structures: large and small, simple and complex, relational and algebraic, and what not. And all these structures, as such, are the subject matter of mathematics. (In principle, of course; in reality mathematics tends to concentrate on certain specific kinds of structures[6].) That some structure is causal (or logical) is a matter not

[2] Michael D. Resnik, *Mathematics as a Science of Patterns: Ontology and Reference* (Oxford: Clarendon Press, 1997).

[3] Of course, he was not the first to articulate this view. Well-known examples are Paul Benacerraf, "What Numbers Could Not Be", *Philosophical Review* LXXIV, 1 (1965): 47–73; G. H. Hardy, *A Mathematician's Apology* (Cambridge: Cambridge University Press, 1992); Keith Devlin, *Mathematics: The Science of Patterns* (New York: Scientific American Library, 1994); or Stewart Shapiro, *Philosophy of Mathematics: Structure and Ontology* (Oxford: Oxford University Press, 1997).

[4] Angela Potochnik, *Idealization and the Aims of Science* (Chicago: University of Chicago Press, 2017).

[5] See Jean-Yves Béziau, "Universal Logic" in T. Childers and O. Majer, eds., *Logica'94: Proceedings of the 8th International Symposium* (Prague: Czech Academy of Sciences, 1994), pp. 73–93. Cf. also Brian R. Gaines, "Universal Logic as a Science of Patterns", in A. Koslow and A. Buchsbaum, eds., *The Road to Universal Logic* (Cham: Birkhäuser, 2015), pp. 145–189.

[6] Though mathematics is nowadays mostly understood as a general theory of structures, it still bears the mark of its previous delimitation as a theory of numbers, so the structures connected with numbers take pride of place among the structures studied by mathematicians. And there are many other reasons to favor certain kinds of structures over others.

of the structure itself, but of the fact that it happens to be displayed by the causal order of the world (or by the order of our argumentative practices, as I will argue below). Hence examining the structures embodied by our logical calculi does not lead us to a universal logic, but rather to (a specific part of) mathematics[7].

So here is an alternative proposal: Logic reflects patterns, but not patterns of the physical (or "metaphysical") world, but rather patterns crucial for human argumentative practices and sedimented in their vehicles, natural languages. Hence in this respect logic is like a natural science, though the patterns it discovers are not straightforwardly causal (though perhaps they supervene on the causal order). Also, logic is not "about" the patterns in exactly the same sense that physics is about its causal ones (see later). However, the similarities are substantial: both physics and logic work their ways from their respective patterns in the real world towards their theories.

What are the patterns that logic is interested in? The most basic relationships underlying them are those constitutive of our practices of "giving and asking for reasons"[8]. These are especially based on the relation of *inference*, which is essential for giving reasons (a reason for a claim must be something from which the claim is inferable), and *incompatibility*, which is essential for asking for them (a claim incompatible with the given one counts as a challenge). Subsuming the relation of incompatibility under the general term[9], we can say that the patterns logic discovers are *inferential*. Thus, for example, we may conclude that, in English, from premises of the shape *A* and *if A then B*, we can always infer *B* (a pattern known as *modus ponens*). More generally, we may say the structure logic focuses on is typically carried by a set of sentences and is constituted by the relation of (correct) inferability among them[10] (plus, possibly, the relation of incompatibility, if we do not want to reduce it to that of inference). Hence, captured most straightforwardly, it might be a set and a binary relation between its

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[7] Of course, it makes a lot of sense to do the mathematics of the structures logic comes to deal with – just as it makes a lot of sense to do the mathematics of the structures physics comes to deal with. The point, however, is that doing mathematical investigations of the structures logic deals with is not yet logic – just as doing mathematical investigations of the structures physics deals with is not yet physics.

[8] A term used by Robert Brandom, *Making It Explicit: Reasoning, Representing, and Discursive Commitment* (Cambridge, Mass.: Harvard University Press, 1994).

[9] We can possibly reduce incompatibility to inference in the well-known way, having ϕ and ψ as incompatible iff every ξ is inferable from ϕ together with ψ. See Jaroslav Peregrin, "Logic Reduced to Bare (Proof-Theoretical) Bones", *Journal of Logic, Language and Information* XXIV, 2 (2015): 193–209 for a general discussion of the relationship between inference and incompatibility.

[10] See Jaroslav Peregrin and Vladimír Svoboda, *Reflective Equilibrium and the Principles of Logical Analysis: Understanding the Laws of Logic* (New York: Routledge, 2017) at Chapter 10.

finite subsets[11]. In this paper, we leave aside the problem of the ontological nature of structures as addressed, for example, by Shapiro[12].

Logic, to be sure, is not interested in all kinds of correct inferences (arguments[13]), it concentrates only on their "core", where the "core" consists of inferences the correctness of which can be ascribed to "logical words", that is which remain correct however we vary their extralogical parts. This, of course, presupposes that the vocabulary of every natural language can be divided into its logical and its extralogical part, where the "logical words" are delimited quite vaguely, as those that penetrate all forms of our discourse (hence are "topic-neutral") and yet are substantial for the evaluation of correctness of arguments. (The vagueness is acceptable for the claim is not that the boundary between logical and extralogical vocabulary is "out there", it is a partly arbitrary expedient of dividing labor between logicians and linguists.)

Thus we assume that any natural language has an inferential structure which is constituted by the facts concerning which inferences are taken for correct and which not. Needless to say, not all such verdicts are univocal and at least some of them are indecisive, so the whole structure is more or less fuzzy. It is, I claim, the work of explicit logical theory to pick out the core of this structure, purge it, complete it and make it definite. We can say, then, that a language contains a proto-logic, which logical theory turns into a logic proper.

II. LOGIC AS A NATURAL SCIENCE?

Let me now point out some aspects in which logic is like a natural science (then I will survey those that make it different). Logic starts from certain data, concerning, in its case, which arguments are correct. From the data it extracts general patterns and presents them as logical theories. Just like natural laws, the laws of logic are not to be encountered in the real world in

[11] But there are alternatives: we can, for example, consider a relation not between sets, but between sequences and so on. Also much less straightforward ways to capture the structure mathematically are possible, including those based on truth-valuations. Elsewhere (Jaroslav Peregrin, *Philosophy of Logical Systems*, New York: Routledge, 2020) I argued that all the artificial languages abundant in modern logic are closely connected with attempts at such capturing.

[12] Shapiro, *Philosophy of Mathematics, op. cit.* What can we say about the specific inferential structures embodied by our languages and targeted by logic (without going into the ontological questions)? One of the early approximations was *Boolean algebra*, that is a complemented distributive lattice. that is a structure in which every element has a complement and every two elements have a join and a meet (distributing over each other). That natural languages display a structure akin to this is indicated by the fact that they contain negations, conjunctions and disjunctions. Needless to say that logic has proceeded to present much more detailed and sometimes also alternative models of natural language.

[13] I use both the term "argument" and "inference" interchangeably for a step from some sentences (premises) to a sentence (conclusion).

a pure form, but arise out of an idealization. Like natural science, logic envisages the patterns it discovers in terms of models, in the case of logic the models acquire the form of "artificial languages", usually equipped with calculi or other tools to help us "compute" what is inferable from that. A natural science like physics also does not usually discover the patterns it deals with in their pure form in the real world (nowhere in the wild, for example, can we observe things dropping to the ground with a uniform and exact acceleration *g*); the models of the world it presents are results of idealization.

It is also useful, I think, to consider a point Potochnik[14] makes regarding science: she claims that the employment of idealizations and models in natural science is "rampant and unchecked". By this she means that "idealizations exist throughout our best scientific representations, and they stand in even for important causal influences" and that "there is little focus on eliminating idealizations or even on controlling their influence" (pp. 57–8). I think that something very similar is true about logic, and that idealizations in logic sometimes remain "unchecked" to the point where this becomes deeply problematic. Of course, it is both useful and legitimate to work with idealizations; but when we forget that they are idealizations and not the ultimate subject matters of our interest, this may become fatal[15].

And one more parallel between logic and natural science: we can see the laws of logic as similar kinds of generalizations as natural laws (despite the divergence in what they generalize – while in the case of natural laws it is facts, in case of the laws of logic it is proprieties – and also in that the laws of logic are reached not by straightforward abstraction, but by a more tortuous process). The laws of nature explain individual events in the world that fall under them just by being the generalizations: a particular object falls to earth because of the law of gravity, namely because this is what objects in general have been found to do. Similarly, the laws of logic explain the proprieties of particular inferences by their falling under general patterns: a particular instance of (MP) is correct just because it is an instance of (MP), namely because instances of this kind have been found to be generally correct. Thus, the laws are not necessarily seen as something that is really there (in the case of logic, somewhere in the human mind or in a Platonic world), they may be seen just in the eye of the beholder (= theoretician).

III. Data for Logic

We assume that some inferences in every natural language are generally accepted as correct, while others are generally rejected as incorrect (and there will also be others not falling into either category). These are the "measurable" data, from which logic works its way to its

[14] Potochnik, *Idealization and the Aims of Science*, op. cit.

[15] Elsewhere (Peregrin, *Philosophy of Logical Systems*, op. cit.) I have argued, in detail, that one of the grave problems of modern logic is that the models have often come to be mistaken for reality.

theories. However, some theoreticians may consider this a non-starter: can the data for logic really be such "sociolinguistic" findings? It is well known that people often reason "illogically" and logic thus must somehow be "above" them in order to correct their excesses.

This leads many philosophers of logic to conclude that the laws of logic cannot be contingent facts of the real world and thus must be discovered by something like an "*a priori* analysis"[16]. However, logic is to help us argue and reason; hence logical theories are useful only insofar as they can be applied to the ways our "giving and asking for reasons" normally proceeds. And arguments, which constitute the steps of our reasoning, are articulated in our natural languages[17].

(Why could we not, instead of in a natural language, reason in an artificial one? The truth is that at least some pioneers of modern formal logic, from Frege on, expected that their artificial languages would replace the natural ones, at least in some restricted contexts. That this has never happened, I think, is no coincidence: it shows that to argue and reason we need natural languages, which have been, during the millennia of their development, so interwoven with our affairs that they have become permeated with meaningfulness in a way we can never mimic in terms of definitions.)

Hence, even if we admitted that we discover laws of logic in a way which has nothing to do with any empirical facts, to put them to use we would have to project them on a natural language in which we formulate our arguments (an enterprise known as *logical analysis* or *logical formalization*). And to achieve this projection ("translation"), we would need to know which words or expressions of the natural language can be taken as counterparts of the logical constants that figure in the laws; hence we would need to compare the meaning of the former with those of the latter. And to determine the meanings of natural language expressions we need to know which arguments in the language are (held for) correct.

Take the rule *modus ponens*

(MP) $$\frac{\phi \qquad \phi \rightarrow \psi}{\psi}$$

and suppose you want to use it to study arguments in a natural language. Suppose that in the language there is a connective *quog* (or, for that matter, *if–then*). How do you tell whether it should obey (MP), that is whether the inference

(MP*) $$\frac{\phi \qquad \phi \textit{ quog } \psi}{\psi}$$

[16] See, for example, Paul A. Boghossian, "Knowledge of Logic", in P. Boghossian and C. Peacocke, eds., *New Essays on the A Priori* (Oxford: Clarendon Press, 2000), pp. 229–254.

[17] Note that I use the term *natural* not in the sense of *everyday* or *ordinary*, but in the sense of *non-artificial*. Hence in this sense even mathematics is carried out almost exclusively in natural languages.

is correct?

Obviously, the only way to find out is to discover the meaning, and especially the inferential role, of *quog*, that is to find out which basic inferences containing it are (held for) correct within the language in question and which are not. Hence the only way to find out whether inferences of the shape (MP*) should be sanctioned is to find out whether they are, as a matter of fact, sanctioned.

Thus we have the transparent $\rightarrow$ and the blurry *quog* (or *if–then*). The use of the former is explicitly delimited and hence it is relatively easy to decide what is the contribution it brings to the sentences containing it; the use of the latter, by contrast, is difficult to encompass. On the other hand, however, the former is mastered only by a few logicians (plus perhaps a few other knowledgeable laypersons), while the latter is a possession of all speakers of the natural language in question. Hence while we can easily argue using *if–then* (or, presumably, *quog* if we were to become competent speakers of the hypothetical language containing it), arguing in terms of $\rightarrow$ is usually possible only on the background of the assumption that it is a kind of shortcut or substitute for a natural language connective.

To avoid misunderstanding, of course the correctness of all inferences is not reducible to public agreement. However, there are some basic inferences which cannot hold for any other reason than agreement. Take the inference from *A and B* to *A*, which, I assume, is accepted by every competent speaker of English. Could it be *wrong*? What would it mean? If it were not to hold, then *and* would not be the kind of conjunction it is now (perhaps it would be a disjunction, or something else?), but there is no sense in which this would be wrong. (What, of course, is wrong is an individual violating such an elementary inference accepted by the bulk of speakers.)

This amounts to the claim that understanding logical words involves mastering the elementary inferences that govern them. (I would be prepared to claim plainly that the meaning of the words *are* their inferential roles, thus subscribing to a wholesale inferentialism[18], but the defense of this standpoint would take me way beyond the boundaries of the present paper[19].) But it would be hard to imagine a change of inferential role of an expression without a change of its meaning – meaning and inferential role appear to be intertwined to the point vindicating the notorious claim of Quine: "change of logic, change of subject"[20].

[18] Of the kind of Brandom, *Making It Explicit*, op. cit. See also Jarred Warren, *Shadows of Syntax: Revitalizing Logical and Mathematical Conventionalism* (New York: Oxford University Press, 2020) for a recent fierce defense of logical inferentialism.

[19] See Jaroslav Peregrin, *Inferentialism: Why Rules Matter* (Basingstoke: Palgrave, 2014).

[20] Willard Van Orman Quine, *Philosophy of Logic* (Cambridge, Mass.: Harvard University Press, 1986) at p. 80.

Which inferences are the *elementary* ones? They are elementary in the sense of not being reducible to any simpler inferences and as we have just seen, they help individuate the meanings of the logical words. I think that Gentzen[21] managed to pick up the most basic ones. But still we must take his collection of rules (constituting his system of natural deduction) as already a *model* of the logical vocabulary of natural language, the inferential roles of logical expressions of natural language are usually more complex. Take conjunction, as the simplest of them. Gentzen articulates one introduction and two elimination rules, and logicians would generally agree that this captures the core of the functioning of such expressions as the English *and*. Yet the functioning is usually more nuanced: for example *and* is sometimes taken to express time succession, so that *He went into politics and became rich* is not taken as synonymous with *He became rich and went into politics*[22].

Once we find out that the inferential role of *quog* (or *if–then*) is close enough to that of $\rightarrow$, we may use the machinery of a logical calculus to tell us which more complicated arguments containing *quog* are correct – and this is certainly something useful. (Just like in natural science: if we find out that a phenomenon is closely enough captured by a model, we can use the model to learn something about the phenomenon.) However, first we must find out that *quog* really can be approximated by $\rightarrow$.

The upshot is that if we want to assess arguments in a natural language, we must be clear about what its expressions mean, especially which elementary inferences they support. We have used the eerie *quog* to indicate that we are dealing with an exotic language, about correct inferences of which we know nothing before we do some sociolinguistic research. But it is important to realize that the situation is not significantly different when we deal with a familiar language, like English. (Perhaps we can paraphrase Quine and say that this establishment of logic "begins at home"[23].) True, here we may spare ourselves some research because we know (or think we know) which inferences are correct; but this is merely a shortcut on the route which is otherwise the same as in the case of a foreign language: it is only when we know what the logical words of a language mean (which involves knowing which basic inferences containing them are correct) that we can correct errors involving these words.

Can there be a language – perhaps a language of thought – for which it would be otherwise? A language the correctness of inferences of which would be a matter of some pure intuition, of some *a priori* knowledge, and which thus would elude the methodology just sketched? Well,

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[21] Gerhard Gentzen, "Untersuchungen über das logische Schließen", I and II. *Mathematische Zeitschrift* XXXIX (1935): 176–210 and 405–431.

[22] Not that more adequate models are impossible. The aspect of *and* just mentioned, for example, is taken into account by conjunction as captured within dynamic logics (Paul Dekker, *Dynamic Semantics*, Dordrecht: Springer, 2012).

[23] When discussing his "indeterminacy of translation", Quine (*Word and Object*, Cambridge, Mass.: MIT Press, 1960) uses this phrase to urge that even our first language must be interpreted.

even if there were such covert language, to use it to assess the correctness of arguments, we would still need to "translate" it into the overt language in which we formulate the arguments. So we would still need to do the empirical research.

Can we bypass natural language and directly use its artificial refinement? We have already seen that as an artificial language is merely a *simulacrum* of a natural one, this is not really possible. An artificial language can help us understand the workings of a natural one (in the way models do) and hence improve the ways we can argue, but it is not a self-standing language.

Hence it would seem that what we do is a kind of idealization leading from some basic data to an explicit system, a journey that is not utterly dissimilar to idealization in natural science. Only this process should not be thought of as concealed within an individual mind, as it may seem to be when we speak about an "*a priori* analysis". It should be seen as carried out in the open so as to be under intersubjective control. Intuitions should be replaced by data regarding which arguments are held for correct in a given community and the idealization should be governed by the methodological canons that are common in science.

To sum up, there are languages that can be used to argue and reason, natural languages, and languages that are invented and constructed by us, which are used as tools of their analysis (and, possibly amendment). The correctness of the most basic arguments of the former is yielded by the *de facto* attitudes of their users (of course, without being employed by them they are just empty sounds or scribbles, no semantic or logical properties are breathed into them by a god), while that of the latter is determined by the definitions formulated by their creators. To use an artificial language as a model of a natural one, we need to ascertain that the logical properties of the former align (to a sufficient extent) with those of the latter.

IV. The methodology of reflective equilibrium

Elsewhere I – and my co-author – argue that the most fitting description of the actual methodology of logic is based on the concept of *reflective equilibrium*[24]. Is this what differentiates logic from natural sciences?

Not really. True, the term 'reflective equilibrium', as introduced by Rawls[25] to characterize the specific methodology of ethics, is certainly far removed from natural sciences. (An earlier similar view on logic was presented, though not under this name, by Goodman[26].) But the

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[24] See Peregrin and Svoboda, *Reflective Equilibrium and the Principles of Logical Analysis*, op. cit.

[25] John Rawls, *A Theory of Justice* (Cambridge, Mass.: Harvard University Press, 1971).

[26] Nelson Goodman, *Fact, Fiction, and Forecast* (Cambridge, Mass.: Harvard University Press, 1955). See Peregrin and Svoboda, *Reflective Equilibrium and the Principles of Logical Analysis*, op. cit., and

ordinary methodology of natural sciences is itself sometimes portrayed as following a reflective equilibrium. Thus, for example, Cummings[27]: "As a procedure reflective equilibrium is simply a familiar kind of standard scientific method with a new name."

Neither logic nor natural sciences find the patterns they aim at articulating displayed in pure form within their data; instead, they form their theories are to fit the data, and at the same time there is a degree of "consolidating" the data to fit the theories. Hence this still fails to drive a wedge between logic, on the one hand, and natural sciences on the other. Logic still looks "non-exceptional"[28].

But on closer inspection, we can see that there are differences. We saw the most basic idea behind *reflective equilibrium*, present within the methodologies both of natural sciences and of logic, is that working towards a theory given some data we not only form the theory to fit the data, but also adjust the data to fit the theory. (The adjustment, needless to say, does not entail forgery, but rather, for example, explaining away outlying values by fine-tuning the conceptual framework so that they are rendered irrelevant[29].) However, the way this works within logic is not quite the same as in the case of natural sciences.

One difference is that if we consider the data for logic as the arguments generally taken for correct, then we need a lot more "consolidation" than in natural science. Another thing is that the data may turn out to be contradictory, which is unacceptable, so we need to "doctor" them to do away with the contradictions. Speakers of a language might for example accept, for some sentences ϕ and ψ, the inference

$$\frac{\phi \qquad \phi\ \mathit{quog}\ \psi}{\psi}$$

and simultaneously accept the sentences ϕ and $\phi\ \mathit{quog}\ \psi$, but reject the sentence ψ. This would lead them to both accept and reject ψ, which is hardly feasible. Or, less trivially, the speakers might hold that, for every ϕ, ψ, and ξ, both the inferences

$$\frac{\phi \qquad \phi\ \mathit{quog}\ \psi}{\psi}$$

Georg Brun, "Reconstructing Arguments: Formalization and Reflective Equilibrium", *Logical Analysis and History of Philosophy* XVII, 1 (2014): 94–129 for more details.

[27] Robert Cummings, "Reflection on Reflective Equilibrium", in M. M. R. DePaul and W. Ramsey, eds., *Rethinking Intuition: The Psychology of Intuition and its Role in Philosophical Inquiry* (Lanham: Rowman & Littlefield, 1998), pp. 113–27, at p. 113.

[28] In the sense of Ole Thomassen Hjortland, "Anti-exceptionalism about logic", *Philosophical Studies* CLXXIV, 3 (2017): 631–658.

[29] Thus, if we understand *fish* as everything that lives in water and have the hypothesis *No fish is a mammal*, then the discovery of whales may not only make us reject the hypothesis, but also redefine the term *fish*.

and

$$\frac{\psi \quad \psi\ quog\ \xi}{\xi}$$

are correct, but their composition

$$\frac{\phi \quad \phi\ quog\ \psi \quad \psi\ quog\ \xi}{\xi}$$

is not, while at the same time they may want to compose inferences in an intuitively standard way, which would be captured as maintaining that (for any sequences Σ and Γ of sentences)

if $\frac{\phi \quad \Gamma}{\psi}$ and $\frac{\Sigma}{\phi}$ then $\frac{\Sigma \quad \Gamma}{\psi}$

Then we cannot accept all these findings as data; and we must retract at least some of them (by inspecting which one the speakers themselves would tend to sacrifice).

A further point is that the very fact that logical theories present norms applicable to the ways people normally reason may lead people to modify their ways of reasoning to abide by them. Thus if logical theories put forward that ψ is correctly inferable from ϕ and ϕ *quog* ψ, this may have some influence on the views held by competent speakers. Perhaps originally this was merely an idealization, and the actual implicit rules governing *quog* were slightly more complicated (or not exceptionless), so that it is only after its incorporation into a logical theory that it holds literally. Thus, while models in natural science do not influence what they model, theories concerning human actions, including logic, may do so.

Additionally, there will be a large grey zone of arguments which are neither clearly correct, nor clearly incorrect, and theories can fill the gap more or less deliberately. Maybe the correctness of the inference

$$\frac{\psi}{\phi\ quog\ \psi}$$

is accepted by part of speakers and rejected by another part; then the theoreticians may rightfully push through one of the options.

Hence the process of zooming in on an equilibrium within logic is a process more complex – and perhaps we can say creative – than anything we can find within the methodology of natural science.

V. THE NORMATIVITY OF LOGIC

Like a natural science, logic captures certain patterns present in a matter-of-factual domain, and like a natural science, it produces theories based on these findings and produced by way of a process in connection with which we can talk about reflective equilibrium. But, we saw, there are also differences which are not unsubstantial. In this section we point out that the differences have generally to do with the fact that logic, unlike natural science, is *normative*. That even the reflective equilibrium is achieved in a different way in logic than in natural science is a consequence of this fact.

The normativity of logic that is responsible for the differences between it and physics comes in two flavors. The first comes to the open when we see that while the target of scientific models are natural facts, those of the logical models are, as we already saw, *rules* (*de facto* governing our practices of giving and asking for reasons), hence "normative facts". Thus, when stating the rule

(MP*) $\dfrac{\phi \qquad \phi\ quog\ \psi}{\psi}$

we are not claiming that the speakers of the language in question, as a matter of fact, tend to infer from ϕ and ϕ *quog* ψ to ψ, but that this is what they *hold for correct* (which is manifested in their tendency to correct those not doing so, and their agreement with those who do concur). Thus, it is not a matter of regularities of the speakers' inferential behavior, but rather of the regularities of their "normative attitudes" to the practices[30].

Now, it is clear that a fact concerning a mathematical structure can be seen as capturing a worldly fact – insofar as we see the structure as a model of some worldly entity. Thus a fact, such as that ψ is inferable from ϕ and $\phi\rightarrow\psi$ in the logic that we favor, can be seen as capturing a fact concerning our natural language – insofar as we see the logic as a model of the language. Moreover, the mathematical fact can be seen as capturing a *rule* or a *propriety* – hence the worldly fact captured by ψ being inferable from ϕ and $\phi\rightarrow\psi$ can be a rule: an English sentence ψ, as a matter of fact, is correctly inferable from sentences of the shape *if* ϕ *then* ψ and ϕ.

However, this alone would not necessarily make for a really substantial difference between logic and physics – if the "normative facts" were taken to be just kinds of facts, namely facts about the existence or acceptance of some rules. But this is not the case: logic does not usually present its theories as descriptions of facts (namely as reports that something counts somewhere as a rule, or somebody reasons in some way), but as *rules*, as instructions how we *should* reason. This, then, constitutes the second flavor of the normativity of logic.

Thus, the normativity of the "normative facts", which are the data of logic, as if "precipitates" into its theories – logic is not a *description* of inferential rules, but rather it is its *explicitation*. Hence, if we reach the conclusion that the inference (MP*) is correct, it is not to be read as a report about what the speakers of the language in question, as a matter of fact, do, nor what they hold for correct, it is an articulation of what they *should* do. (The corresponding rule was

[30] Peregrin, *Inferentialism*, op. cit., at §4.4.

already plus/minus present – implicitly – in the language; and the above argument scheme makes it explicit.) Similarly with the general (MP): it is supposed to tell us how to reason (of course, *modulo* the projection of $\rightarrow$ on the language in which the reasoning happens).

So the common denominator of the difference can be dubbed "normativity", though it comes in two different varieties. In this sense logic, in relation to science, *is* "exceptional". Hence in this way we have reached a stance that can be called *moderate anti-exceptionalism*[31]. Is there any substantial variety of normativity missing from this picture? Should we perhaps further ask which kind of logical operator it is *correct* to reason with – should we use classical, intuitionist or perhaps rather paraconsistent negation?

I do not think so, for as we already saw, we can reason only with a natural language (typically our mother tongue). We cannot reason with artificially produced symbols. Hence the question which kind of negation logicians have produced is not relevant. We can only ask which artificial negation is better suited to capture the negation we do reason with (such as the English *not*), and perhaps whether our natural negation can and should be fine-tuned in the direction of some of the negations.

However, there are still a few lingering questions raised by this stance, which we will consider now.

VI. Is logic empirical?

We have stated that logic is based on empirical data regarding which inferences are taken for correct. Does this not lead to the conclusion that logic is empirical? And does it not then lead to a notion of logic parallel to the ill-reputed Millian notion of mathematics, according to which its truths are simply empirical generalizations? This, it seems, would hardly be acceptable.

But this is not the case. The important point is that though logic starts from the findings concerning which inferences are taken for correct, these findings are not its subject matter. (In other words, as we anticipated already in the beginning of the paper, logic is not *about* its normative patterns in the same sense in which physics is about its ones.) Logic is not an empirical theory of sociological facts, of facts concerning the verbal habits or dispositions of speakers of various languages. Logic is not even an empirical theory of facts concerning the dispositions of speakers to approve or disapprove of the verbal habits of other speakers – though these facts are essential for it. We have seen that logic is *normative*: it is about how speakers of the languages *should* reason in the languages.

[31] Jaroslav Peregrin and Vladimír Svoboda, "Moderate anti-exceptionalism and earthborn logic", *Synthese* CXCIX, 3–4 (2021): 8781–8806.

What is it, then, that logic is *about*? The title of this paper might make one expect that it is about certain patterns – not about patterns as such (like mathematics), but about the patterns that are to be discovered in a certain realm (like physics). But even this is not accurate, for as I have stressed, logic is not a *description* of the patterns we can find in human argumentative practices, it is a canonization of the *rules* underlying the patterns. Thus there is no question of its "reliability" in the sense of its adequacy to a realm it captures, in the sense of Schechter[32]: "How is it that our logical beliefs match the logical facts?"[33] – there are no "logical facts" beyond the empirical facts concerning what users of languages take for correct.

Thus, truths of logic are not empirical generalizations, they are statements of proprieties distilled out of the *de facto* proprieties implicit to the linguistic conduct (especially the game of giving and asking for reasons) of the speakers. They are distilled to produce general rules concerning the language in question, and then once more to produce rules for an artificial language which is considered to be projectable on any natural language. The important point is that they are still rules, not empirical statements.

Read[34] suggests that there is a sense in which the laws of logic are "*a posteriori*" or "quasi-empirical" (in the sense of Lakatos[35]):

> A logic is not just a formal theory: any logic worth the name comes, like any decent scientific theory, with bridging principles which connect it with practice – in the case of logic, with our reasoning practices. So not only can there be logical falsifiers of a logic (...), as of any theory, but also heuristic falsifiers, implications for our reasoning practices which may show their untenability a posteriori.

Thus logical theory, though not an empirical theory, must accord with certain empirical facts – in pain of being useless. So the theory is not merely a theory, but at the same time also a technology – a technology helping us to reason, and especially to find a common language useful for reasoning[36].

This is also connected with the question of logical pluralism, which has become the subject of many recent discussions. The fact that logical theories, as I conceive them, are based on

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[32] Joshua Schechter, "Could Evolution Explain Our Reliability about Logic?", *Oxford Studies in Epistemology* IV (2013): 214–239 at p. 214.

[33] See Justin Clarke-Doane, *Morality and Mathematics* (Oxford: Oxford University Press, 2020) for a thorough discussion of this problem.

[34] Stephen Read, "Anti-exceptionalism about Logic", *The Australasian Journal of Logic* XVI, 7 (2019): 298–318 at p. 314.

[35] Imre Lakatos, "A Renaissance of Empiricism in the Recent Philosophy of Mathematics?", in *Mathematics, Science and Epistemology: Philosophical Papers 2* (Cambridge: Cambridge University Press, 1980), pp. 24–42.

[36] See Jaroslav Peregrin and Vladimír Svoboda, "Logica Dominans vs. Logica Serviens", *Logic and Logical Philosophy* XXXI, 2 (2022): 183–207.

"data", which appear to be simply "given", may appear to indicate that the resulting picture of logic is strictly monistic. But not so.

Remember what comprises the data on which I claim logic is based. Some elementary inferences in any natural language, I maintain, are determined (as correct/incorrect) by the agreement of the bulk of competent speakers, and are thereby nonnegotiable. But there will be plenty of inferences (including probably also some elementary ones) over which there will be no general agreement – so even the meanings of the logical words will be determined only partially. Logic then works its way from these data to its theory.

The road from the data to the theory will be far from straightforward, and it will allow for plenty of possible alternative routes. We need not expect that every theoretician will arrive at the same result. On the contrary, given that the data are incomplete and that what we want to reach is something not firmly tied to the natural language providing the data, we can expect a variation of resulting theories. This yields a substantial plurality of logics (though it is merely a "shallow" plurality that is a matter of underdetermination of theory by data – not a "deep" one, such as that urged by Beall and Restall[37]).

VII. Do we already need logic to reach the reflective equilibrium?

Let us consider one possible objection, concerning the "doctoring" of the data that logic, in contrast to physics, must resort to in order to eliminate inconsistencies. Who decides what is inconsistent with what? Is it not so that, after all, we require some logic that is not "in" our languages, but – as it were – "above" them? And is *this* not the crucial snag that undermines our efforts to assimilate logic to a natural science?

Well, not really. We have seen that the data logic is based on are not inferences that are actually made, but those that are held for correct. This is a subtle, but an all-important difference. Usually, the inferences one makes are those which one holds for correct; but the two notions can come apart. For example, we can discover that an inference we made was wrong, and especially we can be persuaded that this is so. (Sometimes we can even make an inference we, at the same time, consider wrong – for various reasons, such as to deceive our audience.) This means that what we want to capture are not the regularities of our inferential practices, but rather their rules, which may come down to the regularities of our normative attitudes to the practices.

What I am claiming is that it is these rules implicit in our argumentative and inferential practices which are the only source of any logic, of any notions of inference and incompatibility that we have. This is to say that it must also be the source of any notion of incompatibility we use within the process of aiming at a reflective equilibrium – hence if we reject some verdicts

[37] Jc Beall and Greg Restall, *Logical Pluralism* (New York: Oxford University Press, 2006).

of the speakers regarding inferences because they are contradictory to other verdicts, then it is because they are contradictory *by the own lights of the speakers*. The fact that accepting the inferences

$$\frac{\phi \qquad \phi\, quoq\ \psi}{\psi}$$

and

$$\frac{\psi \qquad \psi\, quoq\ \xi}{\xi}$$

while rejecting

$$\frac{\phi \qquad \phi\, quoq\ \psi \qquad \psi\, quoq\ \xi}{\xi}$$

is contradictory must bear on the verdicts of the speakers themselves. (If they do not admit the contradiction, namely if they insist that all this does "go together", then perhaps the logic of their language is "substructural" and arguments cannot be composed in the standard way ...) That is, after collecting the data we must have something as a "second pass" where the speakers detect that some of their previous verdicts do not "go together" and retract some of them.

It is sometimes argued that if we do not already have a background logic, we can produce any logical theory that may cross our mind and then explain away any inconsistency with the data by tweaking the concept of inconsistency. The situation, of course, is not so simple, for inconsistency is part of the logical theory, so it is not so easy to cancel its concrete case, for this usually means rebuilding the theory. But suppose it would be possible to build the theory in such a way that any *prima facie* inconsistency vanishes.

The situation is vividly caricatured by Woods[38]:

> We should not be able to simply revise away tension by removing our belief in it – after all, it's still there – or denying that our general logical beliefs predict it. Revising logic in this way is rather like hiring a plumber to fix a leaky pipe, then watching them look at the growing puddle of water on the floor and insist the pipe is fine. Even if we believe them, we're gonna have soggy carpet.

However, this picture fits the situation where there is a logic that may be changed, or should be fixed (like the leaking pipe), but there is also a rigid background logic which issues the verdict of incompatibility and which is nonnegotiable (like the verdict that a leaking pipe is

[38] Jack Woods, "Against Reflective Equilibrium for Logical Theorizing", *The Australasian Journal of Logic* XVI, 7 (2019): 319–341 at p. 330.

bad). This is not an apt depiction of the situation of the establishment of logic in terms of aiming at the reflective equilibrium, it rather fits the situation of upgrading an existing logic to a new one (because of the incompatibility of the old one with certain intuitively clear cases). Here a logician considers the possible upgrades either on the background of her own logic which is not up for grabs, or at least leaning on a central part of the logic to be upgraded, ready to sacrifice at most some marginal parts.

In contrast to this, in the situation we are interested in no fixed logic is yet available, though a proto-logic is implicit to the language of the community in question. Zooming in on the reflective equilibrium is the way to make the proto-logic explicit and determinate. Likewise, there is no actor who would proceed towards the reflective equilibrium and who could have his own "background logic" not included within the logic to be established. Nobody is reasoning "we could abandon this rule, and rather accept that one instead …", it is a process of ongoing spontaneous alleviation of "tensions" (such as the tendency to both accept and reject the same claim, or more generally, accepting things that do not feel as if they "go together"), namely doing away with cases where the members of the community would seem to have to accept something they do not want to accept. The "tensions" we later call *incompatibilities* have no specific status.

The same holds for a more general version of the objection, namely that to build any theory we need a logic, hence if logic is to result from a theory-building process, we are in a vicious circle. The answer, again, is that this is no more mysterious than that the usual way to forge a hammer is using a hammer. Logic, as a theory, bootstrapped itself into existence together with our abilities to build theories. Our first rudimentary inferences not only laid the foundations of future logic, but also let us reach some consequences of what we already believed, and thus they represent the rudiments of the ability to build theories; and as real logic emerges out of the groping movement towards a reflective equilibrium, so does the skill of getting all the corollaries of what we already know, namely building theories.

Thus, though it is true that we need the concept of incompatibility to arrive at fully-fledged logical laws (if this is to work in the way urged here), initially there is no need for a fully-fledged concept of incompatibility, we can make do with a rudimentary one – hence the whole process of arriving at incompatibility and the laws of logic can be one of bootstrapping.

VIII. Is logic universal?

Another problem with the proposal expressed here is that the data from which logical theory unfolds always concern a concrete language. Hence it seems that what we arrive at starting from them is a logic of this particular language, whereas logic, it would seem, is universal.

But this is also not a devastating problem. Logic, we propose, aims at a structure, at a system of inferential patterns and inferential roles. This system is not necessarily exactly instantiated by the natural language from which we start, it is a result of idealization. Now the very same structure may turn out to be instantiated by other natural languages – again, instantiated not in the sense of being exactly present, but in the sense of being able to serve as a useful idealized model of the language. And this is what turns out to be the case: it seems that all natural languages we know display plus minus a logical structure. (Every language we know contains a negation, a conjunction, some conditionals ...)

Thus, the logical constant $\rightarrow$ governed by rules like (MP) turns out to be able to act as a useful approximation of connectives to be found across various natural languages. The individual connectives (*quog*, *if–then* ...) may function not exactly as $\rightarrow$, but similarly enough that it is useful to use $\rightarrow$ as their model. (More realistically, "logical elements" of a specific language can be modeled by various complexes of artificial logical constants – just like the article *the* was modeled by the constants of classical logic within the celebrated analysis of Russell[39].)

So both in the cases of physics and logic the result of investigation is a structure that is invariant over individual cases, and which can be employed to study individual cases even if these do not embody it perfectly. There are, needless to say, differences. The most obvious – at least *prima facie* – may be described as follows: Any part of physical reality must obey the laws of physics; and for any deviation from the laws we must be able – at least in principle – to pinpoint the intervening factors which are responsible for this. In contrast to this, we cannot be so certain that any language will display basic logical structures – or at least so it would seem, given that the possible deviantness of human communities seems to be limitless. Moreover, we are not supposed to answer the question why a concrete language embodies the principles of logic in an imperfect way. How big a wedge do these differences draw between the two cases?

Perhaps not so big as it might seem. How do we know that everything in the world will obey the laws of physics? We have never encountered an exception, and this warrants us in supposing that we never will. Here, then, the situation with logic is not so dissimilar: we also believe that any natural language will have a logical structure for we have never encountered a language that lacks it. (True, were we to encounter such an exceptional language, we would be reluctant to call it a "language", which seems to make the thesis immune from refutation; but the fact is that to date we have not encountered anything that lacks a logical structure – namely lacks negations, conjunctions and so on – while being urged to call it a language.) The common denominator of physics and logic is that there is a range of phenomena that share a kind of structure, which is picked up by the respective theories and envisaged in terms of idealized models, and the phenomena are then researched partly via the study of the models.

[39] Bertrand Russell, "On Denoting", *Mind* XIV, 56 (1905): 479–493.

IX. Conclusion: Logic as a science of patterns

Despite the differences we have recorded between logic and natural sciences, I think the parallel between logic and natural science goes quite far – in any case much further than anybody has yet recorded. Especially it goes far enough for us to claim that logic is a science of patterns in a sense not dissimilar to that in which a natural science such as physics is.

Logic can be seen as picking up inferential patterns governing our discursive practices (especially the "game of giving and asking for reasons") and working from them towards a state of reflective equilibrium, where the laws it aims at are explicitly articulated. Due to the normativity of logic, this process is not quite the same as that in which a natural science works its way from its data to its laws. However, there is no reason for it to be seen as proceeding via some mysterious "*a priori* analysis" – it should proceed in the intersubjective space where the sciences are done.

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