How to classify varieties of consequence

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Abstract. Göran Sundholm is well-known for his insistence that logic, to be pursued properly, requires a rich conceptual framework that in current logical theories is often, unfortunately, encountered in an essentially impoverished form. One of the conceptual distinctions he has been constantly urging is that between the various senses of *consequence*. I agree that logic needs a rich conceptual framework, and that especially with respect to consequence many crucial distinctions must be maintained. However, these for me are not quite the same as those urged by Göran (at least not obviously so), and in this paper I explain which I think the distinctions should be and why they are crucial. This, I hope, may lead to comparing notes with Göran; which, I believe, may yield some interesting results.

Varieties of consequence

Göran Sundholm is well-known for his insistence that logic, to be pursued properly, requires a rich conceptual framework that in current logical theories is often, unfortunately, encountered in an essentially impoverished form. One of the conceptual distinctions he has been constantly urging is that between the various senses of *consequence*. Sundholm (2019) proposes establishing the following fourfold distinction[1]:

> Within the interpreted perspective of an interpreted formal language, with respect to two propositions A and B, there are at least four relevant notions of consequence here.
>
> (1) the *implication* proposition A⊃B, which may be **true**
>
> (or even *logically true* "in all variations");
>
> (2) the *conditional* [if A is true then B is true],
>
> or, in other words,
>
> B is **true**, **on condition** that A is true
>
> **under hypothesis** that A is true
>
> under assumption that A is true

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[1] For a similar classification, see Sundholm (2012).

(3) the *consequence* [A $\Rightarrow$ B] may **hold**;

(4) the *inference* [A is true. Therefore: B is true] may be **valid**.

This is a very detailed conceptual segmentation, though I am not quite sure I fully understand all its consequences. I agree logic needs a rich conceptual framework, and that especially with respect to consequence many crucial distinctions must be maintained. However, these for me are not quite the same as those urged by Göran (at least not obviously), and in this paper I explain which I think the distinctions should be and why they are crucial. This, I hope, may lead to comparing notes with Göran; which, I believe, may yield some interesting results.

Language vs. metalanguage

First, there is the distinction between cases of consequence that are *explicit*, in that they are articulated as sentences, and those that are merely *implicit* (which, when we engage a language to talk about the language, becomes the good old Tarskian distinction between what is expressed already in the *object language* and what that can be expressed only in a *metalanguage*). We can also say that it is the distinction between what is the case *within* the language we are considering and what holds *about* it.

Suppose we have, as in classical or intuitionist logic, the operator $\rightarrow$ (I prefer this sign to Göran's horseshoe) that is governed by the deduction theorem, i.e. such that

$$A \vdash B \text{ iff } \vdash A \rightarrow B$$

Then, we may want to say that $A \rightarrow B$ is an approximation of $A \vdash B$ in the object language. At least in case of mathematics, where $\vdash A \rightarrow B$ iff $A \rightarrow B$, we can say that $A \vdash B$ and $A \rightarrow B$ express the same thing (are they (3) and (1) of Göran's classification?) – only in different languages. The former states it in the metalanguage, while the latter states it in the object language. If we want to project the metalinguistic "$\vdash$" into the object language for languages outside of mathematics (where $A \rightarrow B$ may be true without $\vdash A \rightarrow B$ being the case), we may think about the introduction of a strict implication urged by Lewis (1912, 1917).

When we look at this from the viewpoint of the object language itself, the distinction becomes that between what is implicit and what is explicit, what can merely be *done* in the language and what can be *said*[2]. Without the arrow sign, the fact that A entails B can merely be endorsed by using the signs in accordance with this, while once the arrow (governed by the deduction theorem) is in play, we gain the ability to say it.

This is no minor difference, because if the rule is not explicit, we can only follow it (or, as the case may be, violate it); whereas when it can be explicitly articulated, we can make it subject

[2] And of course this is related to the Wittgenstein's (1922) distinction between that which can only be shown and that which can be said.

to our game of giving and asking for reasons, we can give reasons for or against it, we can compare it with other alternative rules and we can, possibly, decide to abandon it.

What is crucial is that the appreciation of this difference can lead us to a distinctive view of the nature of logic, especially of the purpose of logical vocabulary. This view, sometimes called *expressivism*[3], maintains that it is precisely the point of "$\rightarrow$" (as well as other logical constants) to let us *say* what we were only able to *do* before. We humans are characterized by the tendency, as Brandom (1994) puts it, to make the implicit explicit. This is the point of departure of expressivism – the view that the point of logical constants is to make explicit and sayable what before was only implicit and doable. This is an unusual view, because it assigns logical constants (and thereby logic) a purpose; and makes them liable to being judged on how well they serve the purpose[4].

By way of digression, we can also mention the possibility of having not a *connective* making inference explicit, but a *predicate* **Inf** such that the following holds (where $\ulcorner X \urcorner$ is a name of the expression X available within the language in question, perhaps its Gödel number):

(Inf1) if $A \models B$ then $\models \mathbf{Inf}(\ulcorner A \urcorner, \ulcorner B \urcorner)$.

(Inf2) $A, \mathbf{Inf}(\ulcorner A \urcorner, \ulcorner B \urcorner) \models B$

Note that given (Cut), (Inf2) entails

(Inf3) if $\models \mathbf{Inf}(\ulcorner A \urcorner, \ulcorner B \urcorner)$ then $A \models B$,

and hence (Inf1) and (Inf2) entail

(Inf) $A \models B$ iff $\models \mathbf{Inf}(\ulcorner A \urcorner, \ulcorner B \urcorner)$.

But unlike the arrow, this brings about some trouble. (Compare the situation with that of a truth predicate. If we were to explicate the predicate *is true* as a propositional operator,

$$\mathbf{T}A \leftrightarrow A,$$

the situation would be trivial. But once we go for a predicate,

$$\mathbf{Tr}(\ulcorner A \urcorner) \leftrightarrow A,$$

we have all the complex and tricky consequences we know from the work of Tarski and his followers.)

The fundamental difficulty with the truth predicate is that it paves the way to the Liar paradox; and similarly, the inferability predicate leads to the so-called Curry paradox[5]. The problem is that once we have **Inf**, we can form, for an arbitrary given B, the formula

$$\mathbf{Inf}(x, \ulcorner B \urcorner),$$

[3] See Brandom (2000), Peregrin (2008), or Arazim (2016).

[4] See Peregrin (2008; 2014).

[5] See Ketland (2012), Barrio et al. (forthcoming), or Hlobil (2019).

and find its fix point, i.e. a formula A such that

(I) $\mathbf{Inf}(\ulcorner A \urcorner, \ulcorner B \urcorner) \leftrightarrow A$

Now we can reason as follows. (I) yields us

$$A \to \mathbf{Inf}(\ulcorner A \urcorner, \ulcorner B \urcorner).$$

Then, given (Inf2), this yields us

$$A \vdash B.$$

And given (Inf1), we have

$$\vdash \mathbf{Inf}(\ulcorner A \urcorner, \ulcorner B \urcorner),$$

which, according to (I) gives us

$$\vdash A.$$

Hence, as $A \vdash B$, we have

$$\vdash B.$$

And as B was arbitrary, we arrive at the provability of everything.

Natural language and artificial languages

Göran restricts his attention to "formal interpreted languages", but I find it inevitable to include also the distinction between consequence in natural language and consequence in formal languages. My reason is that, though these two phenomena should be clearly separated, many logicians freely fluctuate between them, so much so that it is often not clear what is in their focus.

Hence I think that we must observe the crucial distinction between natural languages, i.e. the languages which are here independently of the will of any one of us and which we can research empirically; and languages we create, by stipulation, and which are thus under our complete control. (In a recent book[6] I argued that it is the confusion of these two kinds of language that underlies very many problems in philosophy of logic.)

There is, for example, the connective *if-then*, which we can find in English (and its equivalents, or near equivalents, in other natural languages). We can only learn its properties, including its inferential behavior, empirically. (To be sure about the properties we would have to research across a representative sample of competent speakers of English, but often authors, who are themselves competent speakers, think it is enough to consult their own

[6] See Peregrin (2020).

intuitions[7].) Thus, there may be disputes whether *if-then* follows *modus ponens* without exception or not[8].

In contrast to this, we have artificial signs, such as "$\rightarrow$", "$\wedge$" or "$\vdash$", which we introduce and equip with their inferential properties at will. We may, for example, *stipulate* that "$\rightarrow$" obeys *modus ponens*. Such artificial constants are then often used as regimentations of expressions of natural language – "$\rightarrow$", for example, is often used to regiment *if-then*.

The trouble, as I see it, is the "unbearable lightness" with which many logicians fluctuate between natural language and its artificial simulacra, for example proving that something holds about "$\rightarrow$" in a logical system and automatically assuming that it must hold for *if-then*.

Consider the so-called "paradoxes of implication": It seems to follow, by elementary logic that "Snow is white" implies "If snow is black, then snow is white". But this has an air of paradox only until we distinguish between

Snow is white

If snow is black, then snow is white

and

White(*Snow*)

Black(*Snow*) $\rightarrow$ *White*(*Snow*)

The first argument counts as incorrect. (At least, I presume no native speaker of English would willingly accept its conclusion.) The second argument, on the other hand, counts as correct – as a result of the meaning which classical logic confers on "$\rightarrow$". The disparity of these conclusions is neither problematic, nor paradoxical. (Together they bring out a reason against regimenting *if-then* as $\rightarrow$.) It is only when we assume that *if-then* and $\rightarrow$ are somehow inherently interconnected that the situation starts to look like a paradox.

Inferences v. inferential rules

Inference, Göran stresses, is an *act*. He writes that such an act may be *valid*; I prefer to use the term *correct* (but this, of course, is just a terminological matter). But however we call it, the fact is that some inferences are correct (valid), while others are not. For example, when I infer

[7] I think that a competent speaker may issue reliable (though not infallible) verdicts regarding the meanings simply because she has learned the language in question (just as somebody competent in geography may issue reliable verdicts regarding names of state capitals). However, trouble arises when some philosophers and logicians conclude that if such "consultation of intuitions" lies in the foundation of the enterprise, then the whole enterprise is ultimately a matter of an "apriori analysis".

[8] See, e.g., McGee (1985).

The streets are wet from *It is raining* and *If it is raining then the streets are wet*, the inference is correct, whereas when I infer *It is raining* from *The streets are wet* and *If it is raining then the streets are wet*, it is incorrect. Why is this?

The reason, I contend, is that the semantics of any language is a matter of inferential rules that instill individual inferences with their proprieties. Thus, the rules of English make it the case that the argument (I1) is generally correct, while (I2) is incorrect:

(I1) $\dfrac{\text{It is raining} \qquad \text{If it is raining then the streets are wet}}{\text{The streets are wet}}$

(I2) $\dfrac{\text{The streets are wet} \qquad \text{If it is raining then the streets are wet}}{\text{It is raining}}$

In this particular case the correctness of (I1) may be seen to be a matter of the connective *if-then* alone, i.e. of the general rule

(MP) $\dfrac{A \qquad \text{If } A \text{ then } B}{B}$

The situation is slightly trickier with (I2). The obvious fact is that

(AC) $\dfrac{B \qquad \text{If } A \text{ then } B}{A}$

is *not* a rule. However, this does not allow us to directly conclude that (I2) is incorrect[9]. In any case, the question is what is the role of rules like (MP) and where do they come from.

The answer to the first question, I insist, is that they are constitutive of the meaning of expressions like *if-then*. Gentzen (1934; 1935) suggested that every logical constant is constituted by its introduction and elimination rules (where there is a sense in which the latter are already "contained" in the former). He articulated the rules for $\neg$, $\wedge$, $\rightarrow$, etc. to mimic their natural language counterparts, such as *not*, *and*, *if-then* as closely as possible, hence there must have been some corresponding rules governing the usage of the latter.

Where did the rules come from? Well, those governing the logical expressions of natural languages came into being as "unwritten rules", during the process of evolution of our language, a process about the exact nature of which we can only speculate. Those governing the logical constants of the artificial languages of logic were stipulated by logicians (sometimes in the effort to regiment the implicit rules governing their natural language prototypes).

What is crucial, and often neglected, is the important difference between a rule and a move according to the rule. (This, admittedly, is strange, for the difference appears to be quite straightforward, but in certain contexts it vanishes from the sight of some logicians and

[9] See Svoboda & Peregrin (2016).

philosophers.) As the rules of the kind of (MP) are constitutive of the meaning of expressions such as *if-then*, the meaning of such expressions can be seen as their role *vis-à-vis* these inferential rules – their *inferential role*. This has been noted by many philosophers and logicians, who have set up the respective programs of "inferential role semantics"[10], "inferentialism"[11] and "proof-theoretic semantics"[12]. Yet this idea is under constant bombardment from those who see it as misguided.

The *locus classicus* of the objection is Prior (1964). He formulates the objection quite unambiguously: "only what already has a meaning can be inferred from anything, or have anything inferred from it". (p. 5). Prima facie, this may sound plausible: we may infer a proposition from a proposition, not a meaningless sound from a meaningless sound. Here it is, however, important to make the needed conceptual distinction.

An inference is a move in a "game" (in the sense of Wittgensteinian "language games"). A move presupposes rules which delimit the space in which the moves are correctly done. Hence moves presuppose rules. So far so good.

The thesis of inferentialism is that meaning is constituted by the rules. Meaning, indeed, is *inferential role*: the role of an item *vis-à-vis* the rules. Thus rules and meanings are two sides of the same coin. Hence to say that individual inferences presuppose meanings is to say that they presuppose rules, which is what inferentialism accepts. Still good.

How come, then, that the fact that inferences presuppose meanings is levelled *against* inferentialism, indeed it is often put forward as a knock-down argument against inferentialism? The reason, I contend, is the failure to distinguish between a rule and a move according to that rule. Prior and his followers confuse rules of inference (which constitute meanings, and predate the moves according to them, i.e. individual inferences) with the inferences (which can be carried out only after the rules and meanings are established)[13].

Let me repeat that it is certainly *not* the case that the inferential rules came into being by being stipulated. Inferential rules developed spontaneously as tools of reasoning as a collective activity, and only later they were made explicit. This does not mean that the rules did not exist before they were made explicit (they existed in terms of the participants of the practices taking the doings of one another for correct or incorrect, in terms of what can be called their normative attitudes); but it means that they cannot completely predate the practice of drawing inferences. The rules and the moves they governed bootstrapped themselves into existence in mutual interdependence.

Anyway, once we make the proper distinction, any air of mystery vanishes, not to mention any alleged *reduction ad absurdum* of inferentialism. Of course inferential rules, and hence

[10] Peacocke (1992), Boghossian (1993).

[11] Brandom (1994), Peregrin (2014)

[12] Wansing (2000), Francez (2015).

[13] See Peregrin (2017).

meanings, precede inferences. And of course this does not contradict the view that meanings are inferential roles – if we understand meanings as creatures of inferential rules, not inferences.

Material vs. logical inferences

There are material inferences, and there are logical ones. (I1) is logical, while

(I3) <u>Fido is a dog</u>

Fido is an animal

(I4) <u>Today is Monday</u>

Tomorrow is Tuesday

are material. Some logicians think that there are no correct material inferences, that if we see (I3) and (I4) as correct, it is only because we see them as tacitly containing additional premises – perhaps *Every dog is an animal* and *If it is Monday today, then it is Tuesday tomorrow* – which convert them into *logically* correct arguments[14].

I disagree. I think that (I3) and (I4) are correct as they stand, as long as the sentences they are composed of mean what they do in English (the alleged tacit premises spell out only parts of the meanings of some of their components). Thus, I think that (I3) and (I4) are correct just as much as (I1), the only difference being that (I1) – in contrast to (I3) and (I4) – is also *logically* correct in the sense that it is correct purely thanks to the logical vocabulary it contains.

True, the boundary between logical and extralogical vocabulary of natural language is somewhat deliberate; but once it is fixed we can draw a boundary between logical and material inferences. If we understand the *logical form* of an expression as the expression with all its extralogical parts abstracted away, then we can say that an argument is logically correct iff it is an instance of a valid logical form, where a form is valid iff all its instances are correct[15].

The doctrine of expressivism discussed above suggests that not only are material inferences as self-contained as logical ones, but also, over and above this, there is a sense in which the material inferences are more fundamental. Logical vocabulary, and hence logical inferences, have come into existence because of the material ones, in particular because of the need to make material inferential rules explicit. As long as the material rule allowing us to infer *The streets are wet* from *It is raining* is only implicit to our practices, we can at most follow it (or, as the case may also be, violate it); while once we make it explicit in terms of the conditional *If it is raining, the streets are wet*, we can give reasons for or against it and thus possibly amend or reject the rule as in appropriate.

[14] This view, of course, goes back to the Aristotelian notion of *enthymeme*.

[15] This terminology was introduced by Peregrin & Svoboda (2016; 2017).

The misunderstandings are further aggravated because logicians sometimes use the term *logically correct* or *logically valid* in a different sense: they say that an argument is logically correct (valid) iff the truth of the premises guarantees that of the conclusion come what may – in any "logically possible" circumstances. All three of (I1), (I3), and (I4) would count as logically correct in this sense. However, (I3) and (I4) do not count as logically correct in our sense – their correctness, in our sense, is a matter of the meaning of the extralogical words, *dog* and *animal*, resp. *Monday* and *Tuesday*.

A speculative genesis of conditionals

The expression of a mode of consequence is a conditional, and the different modes of consequence we have discussed provide for different kinds of conditionals. Altogether we have introduced four dimensions along which modes of consequence can differ. As a result, a conditional may belong to the object language (OBJ) or to the metalanguage (MET). It can belong to a natural language (NAT) or to an artificial one (ART). It can express a specific inference (INF) or an inferential rule (RULE). And finally it can be material (MAT) or logical (LOG). All combinations of these distinctions amount to sixteen cases, which are exemplified in Appendix 1.

We can also think about the "genealogy" of these modes of consequence and their respective corresponding conditionals. Such a "genealogy" reflects how the various modes depend on each other. To a certain extent, this "genealogy" may be seen as a hypothesis about the *de facto* history of the development of human language and of our logical reflection of it; in this respect, however, it has to be taken as purely speculative.

What is the point of departure of any consequence are the material inferences implicit to linguistic practice. All expressions of consequence then come into being as means of their reflection, of increasing sophistication. We will use the sign "☞" to indicate such a purely practical inference. Thus, e.g.,

Hugo is featherless, Hugo is a biped ☞ Hugo is human

indicates that speakers of a language tend to endorse the inference from *Hugo is featherless* and *Hugo is a biped* to *Hugo is human*.

Then, the emergence of basic logical vocabulary allows us to formulate first conditionals making these implicit inferences explicit:

If Hugo is featherless and Hugo is a biped, then Hugo is human

Given this, we have also purely logical inferences and the conditionals expressing them:

If Hugo is featherless and Hugo is a biped, then Hugo is featherless

We can then reflect on the fact that these inferences hold not only for Hugo and, by introducing more advanced logical vocabulary, we can express this using the resources of the object language, e.g. as follows:

Every featherless biped is human

Every featherless biped is featherless

Then, when the reflecting of linguistic practice reaches the point where it starts to employ formal means (of regimentation and abstraction), we can turn all the above kinds of conditionals into their formalized versions:

$(\boldsymbol{Featherless}(\boldsymbol{Hugo})\wedge\boldsymbol{Biped}(\boldsymbol{Hugo}))\rightarrow\boldsymbol{Human}(\boldsymbol{Hugo})$

$(\boldsymbol{Featherless}(\boldsymbol{Hugo})\wedge\boldsymbol{Biped}(\boldsymbol{Hugo}))\rightarrow\boldsymbol{Featherless}\ (\boldsymbol{Hugo})$

$\forall x((\boldsymbol{Featherless}(x)\wedge\boldsymbol{Biped}(x))\rightarrow\boldsymbol{Human}(x))$

$\forall x((\boldsymbol{Featherless}(x)\wedge\boldsymbol{Biped}(x))\rightarrow\ \boldsymbol{Featherless}(x))$

Independently of this, the reflecting is likely to ascend to a metalevel, where we can reproduce the above conditionals, making the language itself into the *object* of our considerations:

'Hugo is human' is inferable from 'Hugo is a biped' and 'Hugo is featherless'

'Hugo is featherless' is inferable from 'Hugo is a biped' and 'Hugo is featherless'

'Being human' is inferable from 'being biped' and 'being featherless'

'Being featherless' is inferable from 'being biped' and 'being featherless'

Combining these two dimensions of sophistication (*viz*. formalization and going meta), we might arrive at, e.g., the following[16]:

$\boldsymbol{Featherless}(\boldsymbol{Hugo}),\ \boldsymbol{Biped}(\boldsymbol{Hugo})\ \vdash\ \boldsymbol{Human}(\boldsymbol{Hugo})$

$\boldsymbol{Featherless}(\boldsymbol{Hugo}),\ \boldsymbol{Biped}(\boldsymbol{Hugo})\ \vdash\ \boldsymbol{Featherless}(\boldsymbol{Hugo})$

$\boldsymbol{Featherless}(\boldsymbol{X}),\ \boldsymbol{Biped}(\boldsymbol{X})\ \vdash\ \boldsymbol{Human}(\boldsymbol{X})$

$\boldsymbol{Featherless}(\boldsymbol{X}),\ \boldsymbol{Biped}(\boldsymbol{X})\ \vdash\ \boldsymbol{Featherless}(\boldsymbol{X})$

All of this is summarized in Appendix 2.

Conclusion

I think that there are many distinctions that must be kept in mind when we try to account for consequence. Göran keeps reminding us of some of them; and I, too, want to point out that without observing such distinctions we may easily get stuck in the muddy waters of wrestling with scary logical strawmen. I am even so insolent to maintain that if we were to keep these distinction clearly in mind, philosophy of logic would be deprived of many of its "unsolvable" problems.

[16] Strictly speaking, what becomes connected by the metalinguistic sign "$\vdash$" should be *names* of the formulas of the object language rather than the formulas themselves. But the usual practice is to work directly with the formulas.

Appendix 1. Survey of modes of consequence

OBJ NAT INF MAT

If Fido is a dog, then Fido is an animal

OBJ NAT INF LOG

If Fido is a dog, then Fido is a dog or a cat

OBJ NAT RULE MAT

Every dog is an animal

OBJ NAT RULE LOG

Every dog is a dog or cat

OBJ ART INF MAT

$\boldsymbol{Dog}(\boldsymbol{Fido}) \rightarrow \boldsymbol{Animal}(\boldsymbol{Fido})$

OBJ ART INF LOG

$\boldsymbol{Dog}(\boldsymbol{Fido}) \rightarrow \boldsymbol{Dog}(\boldsymbol{Fido}) \vee \boldsymbol{Cat}(\boldsymbol{Fido})$

OBJ ART RULE MAT

$\forall x(\boldsymbol{Dog}(x) \rightarrow \boldsymbol{Animal}(x))$

OBJ ART RULE LOG

$\forall x(\boldsymbol{Dog}(x) \rightarrow \boldsymbol{Dog}(x) \vee \boldsymbol{Cat}(x))$

MET NAT INF MAT

'Fido is an animal' is inferable from 'Fido is a dog'

MET NAT INF LOG

'Fido is a dog or a cat ' is inferable from 'Fido is a dog'

MET NAT RULE MAT

'X is an animal' is inferable from 'X is a dog'

MET NAT RULE LOG

'X is a dog or a cat' is inferable from 'X is a dog'

MET ART INF MAT

$\boldsymbol{Dog}(\boldsymbol{Fido}) \vdash \boldsymbol{Animal}(\boldsymbol{Fido})$

MET ART INF LOG

$\boldsymbol{Dog}(\boldsymbol{Fido}) \vdash \boldsymbol{Dog}(\boldsymbol{Fido}) \vee \boldsymbol{Cat}(\boldsymbol{Fido})$

MET ART RULE MAT

$\boldsymbol{Dog}(X) \vdash \boldsymbol{Animal}(X)$

MET ART RULE LOG

$\boldsymbol{Dog}(X) \vdash \boldsymbol{Dog}(X) \vee \boldsymbol{Cat}(X)$

Appendix 2. A speculative genesis of modes of consequence

Hugo is featherless, Hugo is a biped ☞ Hugo is human

BASIC LOGICAL VOCABULARY
(*and*, *if-then* ...)

If Hugo is featherless and Hugo is a biped, then Hugo is human
If Hugo is featherless and Hugo is a biped, then Hugo is featherless

MORE LOGICAL VOCABULARY
(*every*, ...)

Every featherless biped is human
Every featherless biped is featherless

REGIMENTATION OF LOGICAL VOCABULARY
($\wedge$, $\rightarrow$, $\forall$, ...)

$(\mathit{Featherless}(\mathit{Hugo})\wedge\mathit{Biped}(\mathit{Hugo}))\rightarrow\mathit{Human}(\mathit{Hugo})$
$(\mathit{Featherless}(\mathit{Hugo})\wedge\mathit{Biped}(\mathit{Hugo}))\rightarrow\mathit{Featherless}(\mathit{Hugo})$
$\forall x((\mathit{Featherless}(x)\wedge\mathit{Biped}(x))\rightarrow\mathit{Human}(x))$
$\forall x((\mathit{Featherless}(x)\wedge\mathit{Biped}(x))\rightarrow\mathit{Featherless}(x))$

GOING META

'Hugo is human' is inferable from 'Hugo is a biped' and 'Hugo is featherless'
'Hugo is featherless ' is inferable from 'Hugo is a biped' and 'Hugo is featherless'
'Being human' is inferable from 'being biped' and 'being featherless'
'Being featherless' is inferable from 'being biped' and 'being featherless'

REGIMENTATION OF LOGICAL METAVOCABULARY
($\vdash$, ...)

$\mathit{Featherless}(\mathit{Hugo}), \mathit{Biped}(\mathit{Hugo}) \vdash \mathit{Human}(\mathit{Hugo})$
$\mathit{Featherless}(\mathit{Hugo}), \mathit{Biped}(\mathit{Hugo}) \vdash \mathit{Featherless}(\mathit{Hugo})$
$\mathit{Featherless}(X), \mathit{Biped}(X) \vdash \mathit{Human}(X)$
$\mathit{Featherless}(X), \mathit{Biped}(X) \vdash \mathit{Featherless}(X)$

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