LOGICAL FORMALIZATION AND THE FORMATION OF LOGIC(S)

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ABSTRACT
The project of logic as a theoretical tool useful for the sciences and humanities involves, as a crucial step, logical formalization – the conversion of sentences of natural language to formulas of a formal language. But what do we do, exactly, when we do logical formalization? What are the criteria of adequacy of the conversion? In how far is logic normative? The paper offers answers to these central (but surprisingly rather neglected) questions and shows that getting a proper grasp on the process of formalization is important for understanding the nature of logic. The key point is that logic as a theoretical tool manages to consolidate our linguistic – in particular argumentative – practices by means of attaining a specific sort of reflective equilibrium. The paper provides a detailed discussion of the answers to the above questions implied by this understanding of logic.

Keywords: logical formalization, logical analysis, reflective equilibrium.

1. Introduction

Logic is an important discipline that is able to infallibly guide our reasoning and to sort out controversies concerning rational argumentation. This premise is widely accepted and, as a result, hundreds of thousands of students all around the world are required to attend logic courses every year. One of the skills they are all expected to master (at least to some extent) is the skill of translating sentences from their mother tongues into the formulas of certain logical systems – typically classical propositional logic and classical predicate logic (CPL). We can call this ability the skill of formalizing. If their teachers were asked why mastering the skill is an important part of logical training, they would probably be somewhat puzzled: it seems to be all too obvious that it is crucial for putting logic to use and benefitting from its knowledge. Thus, questions like: What is formalization good for? What kind of an ability is the skill of formalization? and What are the criteria

1 This attitude, we are convinced, would be displayed even by those teachers of logic who view logic as part of mathematics and do not particularly care about natural language examples.
that decide who has done formalizing well and who has failed? might seem ridiculously simpleminded – the answers seem to be too trivial. We, however, want to argue that the answers are not at all obvious and that their careful examination is very important. In effect, it can provide a key to answering other questions that appear to be more substantial, like What is the import of logic as manifested in logical formalization? Are verdicts of logic fallible? Is logic a science? and Does logic reside inside of our language or does it come from outside to correct its shortcomings and improve its means?

No matter what goes on in our minds when we reason, it is clear that most of our reasoning, and definitely any reasoning that is to be open to public control, proceeds in a natural medium – a language in the most traditional sense of the word. Our thoughts get shaped while we master language and learn to reason. They – for better or worse – often come out embodied in linguistic expressions whose meanings are elusive and indefinite and these features make the reasoning in their medium difficult to check and control. Logic – the science of rational argumentation – is not designed to create entirely new thoughts (new meanings embodied in specific kinds of sentences), but rather to discipline those that we already have and make them liable to fixed rules. Thus, it turns out that if we want to profit from the achievements of logic we cannot avoid starting in the realm of our language and transform its sentences from their natural medium into the shape in which they can be directly subjected to logical rules, viz. from our natural language to a suitable language of logic. (The results of our logical investigations, of course, can then be utilized only if we project them back into the language that we mastered.) If we fully appreciate this wisdom we will see that, though the questions concerning the nature and criteria of formalization may look close to trivial, they deserve very careful scrutiny. The answering of them can significantly contribute to a better understanding of what we all know or seem to know – what logic is about and how it works. And when speaking about “us” here we don’t mean primarily “us” eternal students of logic, but rather “us” who (at least from time to time) teach logic.2

We do not, of course, want to suggest that we are the first ones to point out the importance of these questions. They are implicit to various problems well-known from the history of logic and hence various relevant considerations

2 Exaggerating a bit, we have a suspicion that most logic teachers facing the question as to what exactly are the criteria that they employ to distinguish a correct formalization from an incorrect one might be similarly puzzled as the notorious bearded man who was asked whether he has his beard beneath or under the blanket when he sleeps. (We, of course, do not suppose that they would have to share a fate analogous to his unfortunate one: that, from the time he pondered the question, he was not able to sleep at all, for he was unable to decide where the beard belongs...
LogicaL formaLization and the formation of Logic(s) are scattered throughout the literature. They concern especially discussions of “logical forms” established as a crucial topic of logic and philosophy in the seminal paper of Russell (1905) and developed in many studies including comprehensive and nearly contemporary treatises like Sainsbury (1991).\(^3\) (Awareness of the significance of these questions is also clearly manifested in the works of eminent scholars working in the zone where the philosophy of language and logic overlap – as, e.g. Quine (1960), Lewis (1970), Montague (1974) or Davidson (1984).) Surprisingly, explicit general considerations are scarce: in fact, the only book-length treatment explicitly devoted to the criteria of logical formalization that we know of is Georg Brun’s (2003) book \textit{Die richtige Formel}. Though Brun’s monograph is impressively thorough and we agree with most of its conclusions,\(^4\) we want here to offer a picture from a somewhat different angle. We will divide the cluster of problems and questions related to the concept of logical formalization into three main parts:

1. What, exactly, is it that we do when we do logical formalization? What is the outcome of the process of formalizing a natural language sentence?
2. What are the criteria of adequacy of logical formalization? Given two formulas which aspire at capturing the logical form of a given natural language sentence, how are we to decide which one is more adequate?
3. What is the role of logic as manifested in logical formalization? Can logic be used to correct natural language and its factual usage, or only to describe and summarize the usage?

\section*{2. Formalization and translation}

Let us begin with the question of what kind of ability the \textit{skill of formalizing} is and what we do when we formalize a sentence or an argument. The way we initially discussed the skill suggested that it is a species of the general skill of translation – the ability we acquire when we master a foreign language. When an English speaker masters German, she becomes able to “Germanize” English sentences and texts as well as to “Englishize” German ones. And it seems that the ability to succeed in formalizing can be compared with mastering a language – by skillful formalizing we manifest that we learned the foreign language of, say, CPL. The analogy may have its limits, but \textit{prima facie} it looks quite plausible.

This analogy, however, raises a serious doubt: If it is adequate, then it seems to imply that the criteria of correct formalization might be similarly vague and disputable as the criteria of deciding which translation from

\(^3\) See also Bar-Hillel (1951), Harman (1972), Preyer and Peter (2002), Cresswell (2003), Pietroski (2009).

\(^4\) Some of the ideas are elaborated on by Brun (2014).
English to German is perfect, which is merely fair, and which is wrong. But then, it would seem, the acceptance of the analogy undermines the picture of logic as the discipline that is (to be) the paradigm of exactness and reliability. Logic is designed to guide our rational selves safely through the perils of our world and guarantee that they will not be led astray by fallacies or bogged down in a morass of puzzles and paradoxes; but, if we admit that the process of putting logic to work is open to similar doubts and challenges as the process of translating one natural language into another, then this picture seems to be in jeopardy.

Before we attempt to address these worries, let us examine the parallel between translating an English sentence into German and translating it into the language of logic – in particular the language of CPL – in greater detail. The parallel faces its limits in (at least) three respects:

1) While in the case of German and English it is relatively easy to find a bilingual speaker who is a native speaker of both languages, there is nobody for whom a logical language would be his first language. (The point is not that there aren’t any people who would master the vocabulary and syntax of CPL perfectly, but rather what a perfect mastering of a language in this case amounts to.)

2) The structure of the language of CPL and the majority of other logical languages seems to be extremely simple compared to natural languages – they consist only of a few elementary expressions and a small number of syntactic rules. The question then is how much of the richness of natural languages we might reasonably want to transfer into them at all.

3) The language of CPL cannot be used as such to say anything, for its “sentences” (formulas) contain parameters or uninterpreted extralogical constants – they are more like sentence-matrices than real sentences.

The first two disanalogies do not pose very serious problems. The fact that there are no naturally born speakers of a logical language may even count as an advantage. While in the case of natural languages there can be controversies among their native speakers concerning concrete translations, in case of artificial languages we can expect that there are experts (especially their creators or the heirs of the creators) who would be in the position of being unquestionable authorities on the subject.

The proviso concerning the simplicity of CPL may be discarded by noting that those logicians who were interested in the interface of logic and natural language did develop logical languages and logical systems that are, as to their complexity, comparable to natural language. Seminal in this respect is the work of Montague (1974), but by now this has grown into a true industry.5 It is, however, worth noting that not all logicians by far want

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5 See van Benthem & ter Meulen (2010) for a detailed compendium of its achievements.
their languages to compete with natural language in respect to complexity – on the contrary, many of them see the simplicity of the languages as a basic virtue.

The third proviso, however, is clearly much more serious. If we take it at face value, we are, it would seem, forced to refuse the whole “translation parallel”: if, on one side, we have sentences of a full-fledged language like English (e.g. *Donkeys have ears*) and, on the other, formulas that are, by themselves, meaningless (\( \forall x (F(x) \rightarrow G(x)) \)), then speaking about translation does not make any sense. We have to conclude that perhaps speaking about translating a sentence of a natural language into a logical language was imprecise to the point of being misguided. Formalizing is perhaps not a kind of translation, but rather a kind of extraction – just as we can extract the grammatical form from an English sentence, we can extract its logical form.

The idea that a logical form is something to be extracted might also resurrect the hope that logic will remain the paradigm of exactness and rigorousness it is often thought to be. Natural language might be vague, fuzzy and disorderly, but this is because its expressions provide for only an overt and imprecise expression of some covert logical structures that are crisp and exact and that form its hidden skeleton. The main problem of this picture is that it is wholly unclear how this sense of “extracting” (viz. bringing to light something hidden in some depth) could be backed up – the praxis of establishing logical forms is much more a matter of weighing various factors against each other than discovering a definite thing that was present “inside” of an expression all along.\(^6\) Hence, we appear to be left with the sense of “extracting” in which we extract grammatical forms – not getting hold of something definite, but rather introducing an order on the basis of the vague and fuzzy facts concerning the expression in question.

The hitherto considerations indicate that we have to choose between two options:

a) Formalization is a process that yields a real sentence – a sentence of an artificial language designed in such a way that it makes the (or a) logical form of the original sentence explicit.

b) Formalization is a process that associates a meaningful sentence with something meaningless – a formula that uncovers (embodies) certain logical form.

Surprisingly, the most influential contemporary accounts of formalization – e.g. the abovementioned accounts of Sainsbury and Brun – look like they

\(^6\) In Chomskyan linguistics, the picture of extracting an existing logical form was given a specific sense: the logical forms we extract exist within our *language faculty*, a specific module of our mind/brain responsible for all our linguistic abilities. But we do not believe that this is viable – see Peregrin (2010b) for details.
want to sidestep choosing one of these two options. They instead opt for an account of formalization that suggests (or seems to suggest) a compromise between them. What they offer as the outcome of formalization is a parametric (hence “meaningless”) formula, which is, however, supplemented by a “key” explaining the link between its parameters and elements of the sentence formalized.

Let us take the simple sentence

\[(S1) \quad \text{Donkeys have ears.}\]

If asked to formalize this sentence in CPL, Sainsbury, Brun (and surely many others)⁷ would provide the formula

\[(F1) \quad \forall x \ (F(x) \rightarrow G(x))\]

supplemented by the key

\[(K1) \quad F: \ldots \text{is a donkey}, G: \ldots \text{has ears.}\]

This key is usually called the correspondence scheme.⁸

Thus we seem to be free to take the result of the formalization either as the parametric formula alone, or the formula plus the key, which, once we conceive the interpretation provided by the correspondence scheme as fixed, makes the parameters practically indistinguishable from extralogical constants, i.e. expressions whose meaning is fixed. This looks like an elegant fix, but it is doubtful that such an astute solution is helpful. In our view, it merely successfully masks a problem that must be faced head on if we seek a clear account of formalization: what those who practice formalization really do or should do. We believe that, though it is in many respects convenient to vacillate between the two accounts of formalization, there is in fact no passable middle way and we cannot but choose one of them. So let us weigh the pros and cons of both choices.

First, let us assume that the correspondence schemes are essential and the couple \((F1) + (K1)\) represents a sentence that is for some reason split into two rows. Obviously, the sentence is not an expression of a purely formal language but rather an expression of a language that could be – using the terminology of Tarski (1933) – called formalized. But is there really any point in using the strange two-row notation? Why shouldn’t we directly use the formula (or “formula”)

\[(HF1) \quad \forall x \ (\text{Is-a-donkey}(x) \rightarrow \text{Has-ears}(x))\]?

⁷ It is difficult to make any general claim about teachers of logic, but we suspect that most of them might well adopt a similar strategy (though usually not explicitly).

The problem with this suggestion is obvious: we would have to explain what kind of formula (HF1) is and to which language it belongs. If the terms *Is-a-donkey* and *Has-ears* are expressions just borrowed from natural language, then (HF1) is not really a formula of any of the usual logical languages. In fact, it is no more a formula of CPL than it is an English sentence. Though it is easily readable for any English speaker acquainted with basic logical symbols, it combines expressions that do not really fit together. It might seem that it would be possible to establish a hybrid language that would combine logical symbols with natural language expressions in the way (HF1) does, but it is not quite clear how this can be done and especially what would be gained from such a project.

It is also hard to accept that, if we take this proposal seriously, we won’t end up with one language of CPL but instead with something along the lines of an English CPL, German CPL or Swahili CPL, i.e. the language of logic ceases to be “international”. It might seem possible to evade this problem by suggesting that the terms *Is-a-donkey* and *Has-ears* are in fact not to be taken as expressions of natural languages, but rather extralogical constants of CPL presented in a “mnemotechnical” way, which would be furnished directly with their meanings (some extralinguistic objects like *concepts* or *sets*). Such a maneuver, however, appears to be hopeless. It is reminiscent of the Leibnizian project of a *mathesis universalis*, a universal language based on a vocabulary of ideas that foundered because nobody had an inkling how it could be really accomplished. Those who are inclined to see formalization as a kind of translation of one language into another would be left with the task of translating English sentences into a language that does not exist and it is doubtful whether it could be brought into being.

3. Formalization without truth conditions?

The above considerations suggest that the picture of logical formalization as a translation from one full-fledged language into another full-fledged language faces fatal problems. What, then, about the conception suggesting that the outcome of formalization of a natural language sentence is a parametric (and thus, by itself, meaningless) formula? Does it not face even worse problems? How could we have any criteria that would allow the assessing of the adequacy of different formalizations if we cannot make use of the evidence we usually peruse when we evaluate translations — especially if we cannot compare the truth conditions of the translations with those of the original sentences?

We want to suggest that even if the result of formalization is a meaningless formula, we do not have to give up the idea that there are some comprehensible (meaning based) criteria allowing us to discern good (correct) formalizations from bad ones, and that the steps from a meaningful sentence to a meaningless formula are not merely brought to us by some mysterious insights. We are convinced that the skill of formalization is based, though perhaps only implicitly, on certain mundane (though sometimes complicated) criteria which we are going to make explicit. True, some of the criteria work partly at cross purposes so that it is necessary to balance them against each other but it is precisely this that leads to the result being what, as we will see, has come to be called the reflective equilibrium.

What would our actual journey from (S1) to (F1) look like? The first step would most probably consist in our rehearsing all the possible readings of the former sentence. It seems to be clear that we may read it as saying the same as the sentence

\[(S1') \textit{All donkeys have ears.}\]

or rather as synonymous with the sentence

\[(S1'') \textit{Donkeys typically (normally) have ears.}\]

or perhaps as

\[(S1''' \textit{Donkeys of some specific (contextually determined) group have ears.}\]

etc.

In view of this, we must make a number of decisions. The first is to decide whether we will accept the sentence as it is as a satisfactory starting point of formalization.\(^{10}\) Refusing to formalize it until it is disambiguated might be one possibility. (But then it becomes a question as to whose task it is to disambiguate it.)\(^{11}\) Another possibility is to try to account for the whole spectrum of meanings: If the sentence is interpreted as saying that donkeys normally have ears, or that it is context dependent, then it cannot

\(^{10}\) In fact, here we come to yet another disanalogy between the processes of translating a sentence (or text) from one natural language into another and its formalization.Normally, translators would not refuse to translate a sentence into another language for the reason that it is ambiguous or its meaning is unclear; they would perhaps try to come up with an analogously ambiguous/unclear sentence of the target language. (If this is not possible, they would probably simply choose one of the meanings/interpretations that come into consideration.) In contrast to this, a logician doing formalization faces the choice between refusing to formalize an ambiguous or unclear sentence or taking formalization as a chance to “improve” on the meaning of the original sentence by providing a result that lacks the problematic features.

\(^{11}\) It is worth noting that in some cases the disambiguation may be close to impossible without employing some of the tools of formal logic.
be formalized in CPL (at least not in a straightforward way); if it is interpreted as saying that all donkeys (without exception) have ears, then we can paraphrase it as

\((S1^\star)\) For every individual it is the case that if it is a donkey, then it has ears.

to make its surface closer to the possibilities offered by CPL.

The next step is then relatively easy – it involves a transformation of this paraphrase into an expression of the “hybrid” kind of language mentioned above, replacing those parts of \((S1^\star)\) that now correspond to the logical constants of CPL by these constants, leaving the rest as it is. In this way, we may get HF1. (Using such expressions is acceptable so long as we take them only as a heuristic means and not as expressions of a well-defined language). In the next step, we disregard the parts inherited from English, i.e. we replace them with meaningless parameters. Thus we arrive at the formula F1. The step during which we form \((S1^\star)\) indicates how we, as a matter of fact, extract the “logical form” – the shape of \((S1^\star)\) already anticipates the structure of (F1). Hence, what we do in this step is seek a paraphrase of the target sentence that would prepare it for the injection of the logical machinery of the logical language we want to employ, and it seems that the articulation of the paraphrase is guided by various and rather complicated considerations.

Thus, by presenting this simple example we have not said anything too substantial about how the process of formalization really proceeds and what kind of insights and criteria logicians employ (or should employ) when they search for the adequate logical form of an English sentence (or argument) in a certain logical language. We have only indicated that the practice of formalization involves preliminary considerations and streamlining and that it often makes use of paraphrasing (“translating” within a language).

No matter how the process actually proceeds in particular cases, it is clear that we need criteria that would help us decide which formalizations of a sentence (argument) in a given logical language are preferable. And, as we suggested, it is equally clear that we cannot make use of the criterion that might come as the most natural one (and does come as such to many people who consider the problem logical formalization): namely, the agreement of truth conditions. In so far as the outcome of the formalization is not a sentence, but rather a formula, no truth conditions can be ascribed to it.  

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12 In practice, searching for a suitable formalization is often a rather complex process and there is no such thing as an algorithm to be used by all and which is suitable for all cases.

13 Some authors (e.g. Baumgartner and Lampert 2009) suggest that formulas can be treated as having truth conditions, in particular that the dependence of the truth value of a
Can we have criteria that are not based on truth conditions? The answer is, in our view, positive. As we suggested, the main point of formalization is to extract – and make explicit – some “logical properties” of the formalized sentences. It is therefore natural to assume that the resulting formula is adequate to the sentence in case its (explicit) “logical properties” match the (implicit) “logical properties” of the original sentence.

In view of the fact that logic is essentially concerned with sorting arguments into correct and incorrect, the crucial “logical properties” should be those that characterize the “behaviour” of the original sentence in (some types of) arguments, i.e. properties that could be dubbed inferential. Knowing the logical form of a sentence is, above all, knowing the correctness/incorrectness of the arguments featuring the sentence, i.e. recognizing its inferential role. Thus, we should base the required criterion on a comparison of behaviours of sentences (or formulas) within arguments (argument schemes).

4. Seeking the best formalization

When we embark on a logical analysis (formalization) using a given logical language, we explicitly or implicitly focus on certain kinds of arguments. We must have an initial idea concerning what kind of arguments should be captured by the logical system that serves as our tool. Thus, if we, as we do in this paper, use the language of CPL, we will not expect that our formalization should demonstrate the correctness of, e.g. the following two arguments:

(A1) \[ \begin{align*}
\text{John is older than Peter} \\
\text{Peter is the father of Jim} \\
\text{Jim is younger than John}
\end{align*} \]

(A2) \[ \begin{align*}
\text{It is impossible that Jane does not know that Nixon resigned in 1974} \\
\text{Nixon resigned in 1974}
\end{align*} \]

The first of these arguments is not \textit{logically} correct, while the second might perhaps be considered logically correct if we accept a certain framework of modal and epistemic logics. What does it mean that an argument is \textit{correct} but not \textit{logically correct}? We take the concept of the correctness of an sentence on circumstances can be mimicked by the dependence of the value of a formula on the interpretation of its parameters. We do not think this is viable – see Peregrin and Svoboda (2013) for details.

14 Of course, we may also think about the “logical properties” to be extracted by logical formalization as a crucial aspect of the \textit{meaning} of the original sentence. A wholesale identification of meaning with the inferential role may then lead to the semantic theory of \textit{inferentialism} (see Peregrin 2013).
argument to be fundamental in the sense of not being reducible to simpler concepts (in particular to the concept of truth). The proficiency of competent speakers of natural languages involves their ability to recognize, at least in relatively straightforward cases, what is a correct argument (a correct inferential step) and what is not. Though many such judgments may be heavily context-dependent, there is something like a context-independent core. Correct arguments can be further divided into logically correct (which hold – put simply – merely by virtue of the meanings of the topic-neutral or ‘syncategorematic’ vocabulary used across all discourses – i.e. no change of the state of the world nor any replacement of their categorematic parts can affect their correctness), analytically correct (which hold merely by the force of the meanings of the words involved – no change of the state of the world can affect their correctness) and factually correct (those which hold only in force of some facts, such as John is in Dublin hence John is in Ireland). Of course, the boundaries between these kinds of correct arguments are not clear-cut.

If we want to assess the adequacy of formalization of a sentence A in CPL in cases that are not entirely straightforward, we should consider (implicitly or explicitly) a sample list of perspicuous natural language arguments that contain A as their non-trivial constituent and that characterize the intended scope of CPL. Let us call the arguments on such a list CPL-relevant reference arguments for A.

Generally, we assume that each logical system is associated with a rectification of a certain part of natural language vocabulary. Inferences/arguments founded on the role of the specific kind of vocabulary (including relevant grammatical means) that the system is supposed to capture constitute its intended scope. This scope should be representatively covered by the reference arguments on which the establishment of the corresponding logic rests. The arguments count as perspicuous when competent speakers find them, upon reflection, clearly correct or incorrect. The reference to the logical system (e.g. “CPL-relevant”) is not needed whenever we view formalization from the internal perspective of a particular logical language. We can, of course, also adopt an external perspective and consider the suitability of a logical language for formalization of a sentence (more about this Peregrin-Svoboda 2013).

15 The fact that an argument is perspicuous does not mean that there is no possibility whatsoever of being mistaken with respect to it. Hence, it may happen (though probably very rarely) that after doing some systematization of arguments we find out that the list of arguments we took for perspicuous, and hence which we took for reference arguments, displays some inconsistency and we will have to discard some of them (which we will do following the principle of minimal mutilation of the whole list).
Thus, when we, for example, want to consider formalizations of S1, we should focus on arguments like

(A3) \textit{All donkeys have ears} \\
\textit{Batu is a donkey} \\
\textit{Batu has ears}

We, of course, should not forget about perspicuously incorrect arguments like

(A4) \textit{All donkeys have ears} \\
\textit{Batu has ears} \\
\textit{Batu is a donkey}

Let us now consider an elementary example indicating how the assessment of adequacy of a formalization can proceed in a particular case. Suppose that three students are given the task to formalize the sentence

(S2) \textit{No grey donkeys are lazy}

and they end up with the following respective proposals:

(FS2a) \( \neg \exists x ((Fx \land Gx) \rightarrow Hx) \)
(FS2b) \( \neg \exists x (Fx \land Gx \land Hx) \)
(FS2c) \( \forall x ((\neg Gx \lor \neg Fx) \rightarrow \neg Hx) \)

How could we decide which of the proposals is to be preferred?

In this case, such a list of relevant reference arguments can contain, e.g. the following (correct and incorrect) cases:

\begin{align*}
\textit{No grey donkeys are lazy} & \quad \textit{Batu is not lazy} \\
\textit{Batu is not grey} & \\
\textit{No grey donkeys are lazy} & \quad \textit{Batu is not grey} \\
\textit{Batu is not lazy} & \\
\textit{Every donkey is a herbivore} & \quad \textit{No herbivore is lazy} \\
\textit{No grey donkeys are lazy} &
\end{align*}

If we now, next to the arguments, put parallel lists consisting of argument forms composed of the corresponding formulas of CPL, the sentence \textit{No grey donkeys are lazy} will be formalized in each of the three proposed ways respectively, and we will get the following table. (For a better orientation we write those sample arguments that are – intuitively – correct in bold font and similarly for the argument forms that are valid in CPL.)
How does this list help us decide which of the proposed formalizations of (S2) is to be preferred? The general answer is obvious: Where we have an intuitively incorrect argument that is rendered as valid by its formalization, or where we have, conversely, an intuitively correct argument that is rendered as incorrect, the formalization is undermined (though not necessarily ultimately). Thus, we can see that the first row does not disqualify any of the three alternative formalizations – the natural language argument is incorrect and all its proposed formal counterparts are incorrect as well. The second row suggests that we have a strong reason to reject the formalization (FS2c). The formal argument in the third column is correct but its natural language instance with (S2) in place of (FS2c) is quite clearly incorrect. Since a logically correct argument form should not have incorrect instances, we have a good reason for rejecting (FS2c). The last row provides a reason both for rejecting (FS2a) and for rejecting the (already rejected) (FS2c). The reason is that they both fail to “uncover” the intuitively logically correct argument belonging to the scope of CPL as an argument with logically correct form. Hence, the winning formalization that we (tentatively) embrace is (FS2b), which was not “disproved” by any of the listed reference arguments.

We can generally suppose that the longer and more variable the list of reference arguments is, the briefer the shortlist of the “successful” candidates will be. Finally, we will choose the best candidate(s) from the shortlist on the basis of auxiliary criteria. (We cannot, of course, be sure that a counterexample will not appear in the future, so the results are always only tentative; but they are no more tentative than other scientific results.)

Let us now conjecture about the general principles governing the analytical process. First, we can say that if the formalization of a sentence leads us to render an intuitively incorrect reference argument as a correct one within the formal language, then this formalization of the sentence is not even a candidate for an adequate formalization. (This is a “soundness”
requirement which appears to be close to a *sine qua non*. Second, if a certain formalization of a sentence leads us to render an intuitively correct argument as also being a correct argument within the formal language, whereas another one renders the argument as not correct, then the former is a better candidate than the latter. (This is a “completeness” requirement that is a more-or-less matter.) In the next section we try to extract true *criteria* out of these considerations.

5. The criteria of formalization

If we generalize from the sketch of a method that we presented in the previous section, we can say that the result of formalization is making explicit the place of $A$ within the inferential structure of its natural language, by means of associating $A$ with a formula of a logical language $S$, the position of which within the inferential structure of $S$ is definite and relatively easily manifested. Hence, with the help of $S$ we construct a “map” of the relevant “inferential surroundings” of $A$, making it possible for us to gain an overview over this “inferential landscape”, thus allowing us to see certain kinds of inferential interrelationships of $A$ with other sentences that would be hard (if not impossible) to discern otherwise.

However, it is crucial to keep in mind that if we try to identify the inferential (sub)structures of a natural language that deserve to be made explicit, we will necessarily uncover a somewhat fuzzy and gappy network of relations among sets (or sequences) of sentences (premises) and individual sentences (conclusions). The inferential structure of $S$ will be, on the other hand, definite, determinate and much simpler.

To be able to formulate the criteria of adequacy of a logical formalization of a sentence that have arisen from the above considerations, we must introduce some terminology. A $[Φ/A]$-formalization of an argument containing $A$ will be a formalization with $Φ$ in place of $A$; conversely a $[Φ/A]$-instance of an argument form containing $Φ$ will be a natural language instance of the form with $A$ in place of $Φ$. Thus, given that $A$ is *All donkeys have ears* and $Φ$ is $∀x(P(x)→Q(x))$, the $[Φ/A]$-formalization of A3 will be (given that the formalizations of *Batu is a donkey* and *Batu has ears* are fixed)

$$
\begin{align*}
\forall x(P(x) & \rightarrow Q(x)) \\
P(a) & \quad \rightarrow \\
Q(a)
\end{align*}
$$

Conversely, A3 will be an $[Φ/A]$-instance of (AF1).

Now an argument form containing $Φ$ is $[Φ/A]$-defeated if it has an intuitively incorrect $[Φ/A]$-instance among the relevant reference arguments.
(otherwise it is [Φ/A]-undefeated). Given this terminology, we can articulate the most fundamental criterion of the adequacy of a formalization, which we will call the principle of reliability, rather succinctly:

(REL) \( \Phi \) is a proto-adequate formalization of \( A \) in \( S \) iff no argument form valid in \( S \) and containing \( \Phi \) is [Φ/A]-defeated.

The other criterion implicit to our proceedings that was envisaged in the previous section is the following principle of ambitiousness:

(AMB) Among the proto-adequate formalizations of \( A \), \( \Phi \) is the more adequate the more argument forms that contain \( \Phi \) and are [Φ/A]-undefeated are valid in \( S \).

The mentioned criteria\(^{17}\) are fundamental, but they do not suffice. To complete a truly comprehensive set of criteria we should add some principles guiding the choice for the cases undecided by the previous criteria. The criteria that are commonly applied can be called the principle of transparency and the principle of parsimony. We can articulate the first principle, for example, in this way:

(PT) (Other things being equal,) \( \Phi \) is the more preferable formalization of the sentence \( A \) in the logical system \( S \) the more the grammatical structure of \( \Phi \) is similar to that of \( A \).

The second principle then can be formulated as follows:

(PP) (Other things being equal,) \( \Phi \) is the more preferable formalization of the sentence \( A \) in the logical system \( S \) the more it is parsimonious as concerns the number of occurrences of logical symbols.\(^{18}\)

The import of the principles should be seen as decreasing in the order in which they have been presented. As we have suggested, the first of them is

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\(^{17}\) The first one is closely reminiscent of the principle (GK) of Brun (2003) resp. (VC) of Brun (2014), but a version of such a criterion appears already in Blau (1977); our formulation is somewhat more cautious. The second criterion is analogous to Brun’s (COM).

\(^{18}\) This may sound somewhat strange, but consider, for example, the sentence “All whales are mammals and nothing that is not a mammal is a whale”, which we are to formalize for the purposes of knowledge representation database. The transparent formalization would be \( \forall x (W(x) \rightarrow M(x)) \land \forall x (\neg M(x) \rightarrow \neg W(x)) \). It clearly makes very good sense to disregard the redundant part and prefer the parsimonious formalization \( \forall x (W(x) \rightarrow M(x)) \). In such a context, we might also completely disregard (PT) and aim at the most condensed formalizations available.

We might also want to prefer parsimonious formal languages – languages that employ a minimal number of different logical symbols (but this is irrelevant here, for in the present consideration we adopt the internal perspective and compare formalizations within a given logical language).
close to a *sine qua non* matter (though keep in mind that this holds only in the realm of the intended scope of the logic in question). The second suggests that the logician should not search just for “the safest” formalization but also for the inferentially most “fruitful” one – the one that makes explicit a higher number of relevant valid inferences than competing ones.\(^{19}\) Both of these principles do not distinguish between very dissimilar equivalent formulas (such as \(A\) and \(A \land A \land A \land \ldots\)), and it is the third one that sieves out the redundancies. The fourth one can then be used to do away even with redundancies that are present within the original sentence. The principles can be given more weight within analyses made for certain specific purposes; thus, especially in the case of the last three, there might be various trade-offs (we might, for example, want to have a regimentation that is not quite transparent if it is exceptionally parsimonious.).

### 6. Formalization as bootstrapping

Now, however, we must return to the various simplifying assumptions that we used on our way from the description of the praxis of logical analysis to our tentative articulation of the criteria.

First, the *principle of reliability* states that we can consider \(\Phi\) as a candidate for the formalization of \(A\) only if \(\text{no}\) argument form containing \(\Phi\) is \([\Phi/A]\)-defeated. In fact, this is not quite realistic. We may sometimes encounter what looks like an invalid instance of an argument form that we hold for valid without putting its validity into doubt. Thus, consider the following argument, which looks, at least *prima facie*, as an instance of (AF1):

\[
(A5) \quad \text{All donkeys have common genes} \\
\text{Batu is a donkey} \\
\text{Batu has common genes}
\]

This is clearly not a valid argument.\(^{20}\) Yet, its existence is not likely to make us reject the validity of (AF1) – we will rather claim that (A5) is, despite appearances, *not* an instance of (AF1). Why? We will probably say something to the effect that the predicate *to have common genes* is not an “individual-level” (but rather “group-level”) predicate and that (AF1) is

\(^{19}\) Keep in mind that we are talking about a “general-purpose” formalization of a sentence. If the task is to formalize it with the restricted aim of showing that a particular argument is correct, we may well make do with some simpler version, doing justice to Quine’s “maxim of shallow analysis” (Quine 1960, p. 160).

\(^{20}\) Some English speakers might not even consider it an argument at all because of the awkwardness of its conclusion.
supposed to work only for the “individual-level” ones. However, how do we tell such an individual-level predicate from a group-level one? Well, we might, e.g. say that it is individual-level if it makes for the valid instance of (AF1). But then we would have a circle: an argument form is valid because all its instances are correct, but to be an instance appears to involve being correct.

Is this a vicious circle? Not necessarily. We think that it only points out that what we see as valid forms is not something which we can directly read off of natural language, but rather that it is something that must be bootstrapped into existence. It is all right to explain away some invalid prima facie instances of an allegedly valid schema provided they can be plausibly taken as something negligible; however, if there is no feasible way of moving them into a marginal position, we must retract the validity of the form.

Similar kinds of bootstrapping, in our view, penetrate the whole enterprise of logical formalization. Thus, we return to another unrealistic assumption we made when we started to look for the criteria of adequacy of formalization, viz. the assumption that the formalizations of all other sentences save the one whose regimentation we are pondering are fixed. Taken literally, it would, of course, again lead us into a vicious circle: if we had to base the regimentation of any sentence on already accomplished formalizations of other sentences, the whole enterprise would never really be able to get off the ground.

And the solution is again, of course, a bootstrapping: we start with formalizations of some simple sentences (taking them as tentative) and use them as stepping stones on the way to the formalization of other sentences. Hence, if we are considering \( \Phi \) as a possible formalization of \( A \) and we find out that some argument form involving \( \Phi \) as a counterpart of \( A \) is valid, whereas if there is a counterexample, we will not only consider dropping the hypothesis that \( \Phi \) is an adequate formalization of \( A \), but will also take into account the possibility of keeping the hypothesis at the cost of dispensing with formalizations of some of the other sentences involved in the counterexample. Again, the process of formalization is in fact a holistic, give-and-take enterprise.

The third simplifying assumption was implicit in our presumption that the logical language that we use for formalization is given and fixed. Any formal language used as the tool of formalization is always more or less Procrustean, and to a certain extent this may be seen as its virtue: it lets us get rid of those elements of natural language that are irrelevant from the viewpoint of argumentation (in the broad sense) and lets us clearly see the relevant backbone. But it might well come to be Procrustean to the extent that it becomes a vice: it makes us neglect or underrate some important feature of natural language. In such a case, we need to ascend to a meta-perspective and look for a more suitable language.
Hence – and this is essential – even the language we use for the formalization must be bootstrapped into existence: to a certain extent, the features of natural language that do not fit into the mould of such language, and of the way we use it to formalize natural language, are tolerable if they can be explained away as irrelevant or marginal. But once this extent is surpassed, it may be wise to give up on the (tentative) logical language and upgrade. (The fact is, the standard logical languages, like those of classical sentential and predicate logic, have come to be taken so much for granted that we often take their adequacy as self-evident and just ignore any discrepancies between them and natural language.)

7. Logic and reflective equilibrium

It is now time to move on to the third group of questions suggested in the introductory part of this paper. The considerations of the previous section indicate that logic is not only striving to get a grasp on the specific kind of patterns that are characteristic for everything we want to call language in a fully-fledged sense (and in this sense it is answerable to how languages really work just as mechanics is answerable to how bodies move), but also has a normative role to play: once it acquires a shape, it assumes the role of a standard which can be used to adjudicate individual cases of argumentation in natural language. As long as logical rules are in force, they decide what is a correct argument. But once requirements for correcting arguments issued by some logical theory become too abundant or too counterintuitive, we must decide to give up on the theory. Hence, we have here the most basic give-and-take. And here is where we think we must see it as a matter of what is aptly called the **reflective equilibrium**.

Both in science and in everyday life we keep acquiring, checking and revising various generalizations. *Metals are electricity conductors. No animal breathes under water. Every fish has gills.* The basic way of acquiring such generalities is induction: we observe a lot of individual cases and form a general thesis. The thesis is, of course, always tentative and subsequent counterexamples may force us to give it up. But when we have a large number of cases and no counterexamples for extended periods of time, we tend to take the thesis as being almost irrefutable; hence, later, when a counterexample does occur, we are reluctant to simply give the thesis up.

21 Take the so-called paradoxes of implication (inferences from \( \neg A \) or from \( B \) to \( A \implies B \)). Argument forms that are adopted as logically correct due to the paradoxical features of material implication have instances that hardly any speaker of English would consider to be correct. Yet, we often find claims to the effect that logic has “shown us” that in fact these natural language arguments are correct.
We check to see whether the counterexample might not be explained away. Especially, we check whether it might only be an apparent counterexample, e.g. the result of an error of measurement. But what if it is not and we still regret giving up a neat generalization?

There remains one more possibility – revising our concepts. The point is that any general thesis is formulated by means of some concepts (metals, electricity, breathing, etc.) and fine-tuning the concepts may tamper with the thesis in such a way that the counterexample might no longer refute it. Suppose that we have the concept of fish such that it encompasses every animal that lives in water and has fins. We conjecture the general thesis Every fish has gills, and for a long time we encounter only fish with gills. Then we come across a whale and our general thesis is in jeopardy. What are we to do? One possibility is to give up our general thesis. Another possibility is to refine our concept of fish and build having gills directly into it. (In this very case we thus save the general thesis Every fish has gills at the cost of making it trivial, analytic; but it is not difficult to imagine less trivial cases.)

It would not make sense to apply this method whenever we encounter a counterexample. However, if we have a very well confirmed general thesis which has already turned out to be useful and only isolated counterexamples, to explain the counterexamples away by tampering with concepts may be reasonable. But, with this, we move into the dangerous proximity of a circle that, as we already saw, might turn out to be a vicious one: a general thesis holds because there are no counterexamples to it, but if we could explain away any counterexample, then, it might seem, anything might be defended as a general thesis. This indicates that the method makes sense only when the thesis is very well confirmed, by lots of positive cases, and the negative cases are only sparse or unimportant – we will consider some different perspectives and search for an equilibrium.

The term “reflective equilibrium” was coined by Rawls, who stresses that searching for this kind of balance is central in the context of ethical considerations (see Rawls 1971). The ultimate origin of the employment of reflective equilibrium (RE) within the philosophy of logic is considered to be Goodman (1955), who, though he did not use the term explicitly, envisaged a scenario of embarking on the laws in logic in precisely the way for which the term has later come to be used:

[D]eductive inferences are justified by their conformity to valid general rules, and that general rules are justified by their conformity to valid inferences. But this circle is a virtuous one. The point is that rules and particular inferences alike are justified by being brought into agreement with each other. A rule is

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22 Well, not really, as explaining away the counterexamples changes the original thesis; hence, we should say any general thesis can be transformed into a modified thesis that holds.
amended if it yields an inference we are unwilling to accept; an inference is rejected if it violates a rule we are unwilling to amend. The process of justification is the delicate one of making mutual adjustments between rules and accepted inferences; and in the agreement achieved lies the only justification needed for either.

And in fact, as some philosophers have pointed out, the process of reflective equilibrium is in no way far removed from the ordinary scientific routine. Thus Cummins (1998) writes:

As a procedure, reflective equilibrium (RE) is simply a familiar kind of standard scientific method with a new name. (...) A theory is constructed to account for a set of observations. Recalcitrant data may be rejected as noise or explained away as the effects of interference of some sort. Recalcitrant data that cannot be plausibly dismissed force emendations in theory. What counts as a plausible dismissal depends, among other things, on the going theory, as well as on background theory and on knowledge that may be relevant to understanding the experimental design that is generating the observations, including knowledge of the apparatus and observation conditions. This sort of mutual adjustment between theory and data is a familiar feature of scientific practice. Whatever authority RE seems to have comes, I think, from a tacit or explicit recognition that it has the same form as this familiar sort of scientific inference.

Resnik (1985, p. 229) describes the specific case of logic in a vivid way:

Once we decide on why we want to model a particular deductive practice by means of a given logical theory, we start with sample cases and fit them in the model (that is, formalize them) and proceed from there to build a larger and larger collection of successful cases. (...) Sooner or later we are bound to encounter anomalies or sticky cases. Then several options are open to us. We can question the application of the model to the case; perhaps, the formalization has gone awry. Or we can question the details of the case; perhaps, we have omitted an implicit premise or have misconstrued one of the premises. Finally, short of renouncing or refurbishing our logical theory, we may dismiss the example as a don’t care case. Whether we succeed with any of these options will be a matter of comparing the success of our logical theory against that of the alternatives to it. Success here is measured in the usual pragmatic terms: the ability of our theory to handle a large number and variety of cases, the simplicity of its account, its fruitfulness for other branches of knowledge, its fit with our prior scientific commitments, and so on.

Reflective equilibrium vs. creation ex nihilo

RE as a methodology of empirical science does not seem to be too problematic; indeed, it seems that there is no other way of reaching a system of robust empirical generalizations than working back and forth between data and tentative generalizations. However, the situation appears to be much trickier if we want to use it to explicate logic and logical laws.
One of the most crucial problems of understanding logic in terms of RE is the fact we seem to need (at least a modicum of) logic to work towards the equilibrium – and if logic were to be only the outcome of RE, we seem to be trapped in a vicious circle. This problem is vividly envisaged by Shapiro (2000), who discusses the proposals of Resnik (ibid.):

Resnik says that the logician tries to “build a logic whose pronouncements accord with [her] initial considered judgements”, and the logician constantly checks if an intuition “coheres with” her other beliefs and commitments. The theory determines when the “data” and the “theory” are in conflict. What is the logic for this? Presumably, at each point in the process, the theorist is to use the logic accepted at that point. The logician is on the ship of Neurath, building that very ship. She uses the logic she is developing in order to modify that very logic.

Hence, imagine the situation when a logician building her logical theory along the lines of the RE methodology faces a situation when some data contradict her tentative theory (perhaps certain concrete arguments that seem clearly incorrect appear to be instances of some pattern held for valid). To reach the RE, she should either explain away the unsuitable data (perhaps by finding ways of explaining why the argument, despite appearances, does not fall under the valid patterns) or revise the theory (modifying the valid patterns), viz. revising the logic. But Shapiro points out that there is a third possibility: if we are free to change our logic, we are free to change it also in such a way that the data no longer come to contradict the theory. And this might be a very cheap way of reaching the RE.

This is a challenge of a sort similar to that which Quine (1936) posed against Carnap’s conventionalist foundation of logic. We cannot, Quine pointed out, assume that we know logical truths simply by knowing (and especially by having stipulated) the meanings of logical constants, for to get from the latter to the former we must apply logical rules which are one side of the coin, the other side of which are logical truths. Hence again the objection is that we cannot see logic as being constituted by a certain procedure, because the procedure already incorporates logic.

The response to this challenge is that explaining logic in terms of RE is not explaining how logic came into being ex nihilo. We must distinguish two senses of “logic”: in one sense, logic is something inherent to our language and it is under our control only in so far as our language is under our control (which is not very much); in another sense, logic is our explicit theory of this something. Logic in the latter sense is something we intentionally build (to explicate/upgrade/replace/account for) logic in the former sense; and it is logic in this latter sense which we claim is a product of RE.

We assume that before we can set out to do the theory of logic, indeed any theory, we must be in possession of some language and this language must incorporate some structures that are logical in a sense that precedes
any logic conceived as an explicit theory. Logical structure of this kind is a condition *sine qua non* for any communication system that is to be called language in a full-fledged understanding of the term.\(^{23}\) It is thus not the case that just about anything can be called logic – if logic is to back up building theories, justifying them, arguing for them, etc., then it cannot be too different from what we call logic as it must be built up on certain structures inherent in natural languages. Hence, Carnap’s (1934) *principle of tolerance* is not feasible if it is interpreted radically as the thesis that logic is essentially (or even purely) a matter of convention.

On the other hand, the “logics” that are implicitly contained in existing languages are to a certain extent and in some respects vague, indeterminate and open-ended – and hence to reconstruct them as more precise, decisive and explicit is a project that is not simple, though it may be greatly rewarding. This process is not just a description of what we can find in language, it is a project involving regimentation, streamlining and extrapolation – but in no case is it a creation *ex nihilo*.

We must therefore agree that engaging in the process of building a logical theory through RE presupposes some “logic”. This core “logic” is not really then a subject of RE considerations; it is, as it were, “transcendent” to them – but not because we would decide to exempt it from them or to immunize it from revision, but rather because its robust presence in our language is a presupposition of any theory building, and hence it is resistant to any revision. (This is not to say that this core logic would be *totally* immune to revision, but that its revision might happen only as a very slow, complex and Neurath-boat process.)

Hence, the *rationale* of applying RE to the foundations of logic consist in the fact that building the logical theory is *in this respect* parallel to building scientific theories: there are phenomena to be described and formulating the theory is a matter of establishing an order within the phenomena, an order which is derived from the phenomena which is, however, not directly contained in them (in a pure form). In particular, understanding logic in terms of RE makes sense even if we assume that some kind of logic is here before we start to do any *theory* of logic – not in a quite articulated and unambiguous form, but is here nevertheless. We assume that we can start to do logical theory only if we already have a language embodying some logical structures, i.e. such that some of the arguments articulable in the language are held for correct and some for incorrect.

\(^{23}\) The question of how languages with logical backbones come into existence is a very different one, which we do not claim to be able to answer in terms of the reflective equilibrium. For the purposes of answering this question, game-theoretical means of the kind employed by Lewis (1969) might be appropriate. See also Peregrin (2010a).
9. Conclusion

In this paper we have tried to show that RE considerations are not crucial only on the macro-level of formation of logical theories but that they also have an important role on the micro-level of particular logical formalizations. We could perhaps say that one of the skills that students of logic improve while getting proficient in formalization is the skill of searching for a reflective equilibrium.

Let us now return to the groups of questions we presented at the end of the opening part of this paper and try to summarize the answers we have given for them up to now.

1. **What exactly is it that we do when we do logical formalization? What do we achieve when we “translate” a natural language sentence into a formula of a formal language?**

   We try to identify the place that the sentence that is to be formalized occupies within the “inferential landscape” of its language. We often proceed by paraphrasing and by “translating” the sentence into a formula of a kind of hybrid language, from which we then can abstract away the (extralogical) remnants of natural language, thus reaching a formula that embodies what is traditionally called the logical form of the sentence. During this process we are guided by the ideal of a maximal possible match between intuitively correct arguments and arguments rendered as valid by means of the logical apparatus of the formal language we use for the formalization. The logical forms are useful in that they make explicit the inferential properties of the formalized sentences and their behaviour within arguments.

2. **What are the criteria of adequacy of logical formalization? Given two formulas which aspire at capturing the logical form of a given natural language statement, how are we to decide which one to prefer?**

   The most basic is the criterion of reliability, and it is supplemented by the criteria of ambitiousness, transparency and parsimony. The criteria usually do not yield anything like the unique logical form; especially the latter three operate on a give-and-take basis. Sticking with the principle of parsimony might, for example, make us use very simple logical tools (like those of classical propositional logic) even at the cost that we will depart further from the grammatical structure of the original sentence (and that they may perhaps even render some relevant intuitively valid arguments as invalid). But even the first criterion is not essential in the sense that it would be absolutely non-negotiable.

3. **What is the role of logic as manifested in logical formalization? Can it be used to correct natural language and its factual usage, or is it only used to describe and summarize the usage?**

   The project of logic as a theoretical tool aims at bringing order to our argumentative practices by means of achieving a specific sort of *reflective*
equilibrium. Logical theories thus have a certain descriptive aspect in the sense that they have to reflect the basic inferential structures of natural language but also a normative aspect in the sense that, once established, they have a (limited) authorization to brand natural language arguments as correct or incorrect. In this sense, logic can be used as a norm that partially corrects usage of natural language where this is needed.

Summing up, we can say that language is the most powerful device acquired by mankind. It not only essentially enhances the possibilities of interindividual communication and coordination, but also considerably enhances the ways we can think and reason. And logical theories provide an essential tool of reflecting the possibilities of reasoning and argumentation, allowing us to see their problematic points and do away with them. But they do not come from a heaven superordinated to our linguistic and cognitive practices; the theories deserve the proud title “logical” only if they accord with the logic inherent in full-fledged languages. Thus, we can see logic as a backbone of any language (worth the name). It is a backbone that is liable to all kinds of scolioses, kyphoses, lordoses, etc., which the logician tries to cure by producing an “ideal backbone” that is to be used as a standard. This backbone, we are convinced, can never be transplanted into the living bodies of our natural languages, but its existence significantly enhances our chances of understanding each other whenever understanding (or sorting out misunderstandings) is essential.

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