BRANDOM'S INCOMPATIBILITY SEMANTICS

Comments on Brandom's Locke Lecture V

Jaroslav Peregrin[*]

Why formal semantics?

Formal semantics is an enterprise which accounts for meaning in formal, mathematical terms, in the expectation of providing a helpful explication[1] of the concept of the *meaning* of specific word kinds (such as logical ones), or of words and expressions generally. Its roots go back to Frege, who proposed exempting concepts, meanings of predicative expressions, from the legislation of psychology and relocating them under that of mathematics. This started a spectacular enterprise, fostered at first within formal logic and later moving into the realm of natural languages, and featuring a series of eminent scholars, from Tarski and Carnap to Montague and David Lewis.

Partly independently of this, Frege set the agenda for a long-term discussion of the question of *what a natural language is*, his own contribution being that language should be seen not as a matter of subjective psychology, but rather as a reality objective in the sense in which mathematics is objective. His formal semantics, then, was just an expression of this conception of language. And many theoreticians now take it for granted that formal semantics is inseparably connected with a Platonist conception of language.

Moreover, the more recent champions of formal semantics, Montague and David Lewis, took for granted that natural language is *nothing else* than a structure of the very kind envisaged by the theories of formal logicians. While Montague claims quite plainly that there is no substantial difference between formal and natural languages ("I reject the contention," he says, 1974, p. 188, "that an important theoretical difference exists between formal and natural languages"), Lewis states that it is fully correct to say that a linguistic community entertains a language in the form of a mathematical structure ("we can say", states Lewis, 1975, p. 166 "that a given language L [a function from a string of sounds or of marks to sets of possible worlds] is *used by* or is a (or the) language *of* a given population P").

However, in the last third of the twentieth century, many (direct and indirect) participants of the discussion of the question *what is language*? underwent a distinctively *pragmatic turn*.

---

[*] This paper evolved from the commentary I gave to Brandom's fifth lecture during his re-presentation of his '06 Locke Lectures in April 2007 in Prague; however, the evolution was so rampant that the paper no longer quite resembles the original commnetary - I hope I have managed to concentrate on what is really vital and left out marginal issues. Work on this paper was supported by the research grant No. 401/04/0387 of the Czech Science Foundation

[1] In the sense of Carnap and Quine, in which an *explicatum* is "a substitute, clear and couched in terms of our liking" filling those functions of the *explicandum* "that make it worth troubling about" (Quine, 1960a, pp. 258-9).

Philosophers of language started to revive American pragmatism, or the ideas of the later Wittgenstein, or Austinian speech act theory, and they began to picture language primarily as an activity – or as a toolbox for carrying out an activity – rather than as a mathematical structure. In response to the Montague-Lewisian conception of language, Donald Davidson, 1986, wrote that "there is no such a thing as language, not if a language is anything like what many philosophers and linguists have supposed".

Bob Brandom's philosophy of language undoubtedly fits with the latter stream. For Brandom, language is first and foremost an activity, albeit a rule-governed one. From this it might be expected that his attitude towards formal semantics would be very reserved. But surprisingly, this does not seem to be the case – what Brandom urges in the fifth of his Locke Lectures is strongly evocative of a formal semantics. Is this viable? Does not the approach to semantics entertained by Montague (1974) or Lewis (1972) presuppose a kind of 'formal metaphysics' which is utterly at odds with pragmatism? Can a Brandomian style inferentialist really embrace formal semantics?

To sort things out, I will concentrate, in this paper, on the following three questions:

(1) Is a formal approach to semantics compatible with pragmatism and inferentialism?

(2) If so, what kind of formal semantics is useful from the inferentialist perspective and of what good can it be from this perspective?; and

(3) is Brandom's *incompatibility semantics* a suitable kind and does it bring us some new insights into the phenomena of meaning and language?

My answer to question (1) is, as expected, positive. For some time now, I have been urging a reconciliation of pragmatism and formal semantics – see esp. Peregrin (2001; 2004)[2]. I am unsure about how far my reasons for giving this answer overlap with the reasons that made Brandom produce his incompatibility semantics; but I think that writing this paper might be a good way to find out.

I personally think that it is vital to appreciate the distinction between claiming that language *is* a mapping of expressions on objects and saying that it is useful to *model it as* a mapping of expressions on objects. The former claim commits us to a radically limiting statement about what meaning is and how an expression can come to have it. Elsewhere (*ibid.*, §8.4) I compared this to the relationship between the Bohr model of the internal workings of an atom (the tiny sun-nucleus orbited by even tinier planet-electrons) and its actual innards. It is clear that the atom's innards are not identical to what the Bohr model shows; but neverthless it is useful to model it in this way. Why? Because, firstly, what the inside is precisely like anyway defies description by the usual means of our language; and, secondly, it renders the inside more comprehensible to us (at the cost of oversimplifications). In short, it provides for what Wittgenstein (1953, §122) called, in the case of language,

[2] My urging of a 'structural', rather than 'metaphysical' reading of formal semantics goes even much farther back, to Peregrin (1995), a book whose subtitle was *Formal semantics without formal metaphysics*.

perspicuous representation [*übersichtliche Darstellung*]; or what Haugeland (1978) called *structural explanation*.

An important point is that whereas saying that a language is a set of labels stuck on objects (a claim that is often ascribed to anybody doing formal semantics) involves saying that the way an expression acquires meaning is by becoming a label, saying that language can be modeled as such a set of labels involves no such commitment. Especially, the latter is fully compatible with seeing meanings as roles: though an expression would acquire such a role by means of being engaged within a praxis, there is no reason not to envisage the roles by means of a model in which the expressions are put side by side with their roles, which are encapsulated into some kind of formal objects.

What does it take to be a model of language?

Given that there is a point in doing a logico-mathematical model of language, we should ask ourselves what kind of model we want, what such a model is to show us, and how are we to assess its adequacy (and hence 'reasonableness'). This brings us to the above question (2). To answer it, let us consider the standard format of the definition of the formal languages of logic, which are to serve us as our models.

Such a language usually has three parts (one of the latter two might be missing). First, there is *syntax* (*proper*), the delimitation of well-formed formulas (and more generally well-formed expressions). Second, there is an *axiomatics* (*logical syntax*, in Carnap's phrase), the delimitation of the relation of inference among wffs and/or the set of theorems. Third, there is a (*formal*) *semantics*, the delimitations of acceptable assignments of denotations to expressions, which ground the relation of *consequence* and/or the set of *tautologies*[3].

If we want to use such a language as a model of an actual language, then which of its features should be aligned with corresponding features of the natural language to be modeled? It is clear and uncontroversial that the first component, the delimitation of wffs, should somehow correspond to the grammar of the language to be modeled. Of course, the correspondence may be very indirect (we may want to have the grammar of the formal language significantly simpler than the grammar of the natural one[4]) – the important thing is

[3] It is also important to realize that such languages may be (more or less) parametric, i.e. they may be mere language forms rather than real languages. When, for example, we are doing logic, we are interested only in logical constants, and treat the rest of the vocabulary as parameters – we take into account all possible denotations of these expressions and thus gain results that are independent of them. But the boundary between constant, i.e. fixedly interpreted, expressions and those that are parametric, i.e. admit variable interpretations, can be drawn also at other joints. Hence it is always important to keep an eye on the extent to which the language we are considering is a true (fully interpreted) language and the extent to which it is only a language scheme.

[4] A paradigmatic example of the discrepancy between the grammar of the regimented natural language and that of the regimenting language of logic is the case of variables of the languages of standard logic. Originally, variables were introduced as basically 'metalinguistic' tools; and it is well known that logic can make do wholly without them (Quine, 1960b; Peregrin, 2000). However, it has turned out that we reach a great simplification of the grammar of the logical languages if we treat variables as

that we should be able to say which wff (if any) corresponds to a given sentence and which sentence corresponds to which wff.

Many logicians would say that the point of contact between the real language and the model is the semantics – that the denotations of the expressions of the formal language assigned to expressions should *correspond to*, or *model*, or *capture*, or simply *be* the meanings of the corresponding expressions of natural language. Thus, classical logic maps '∧' on the usual truth table, the story goes, because the corresponding truth function is the meaning of English 'and', which gets regimented as '∧'. This amounts to saying that the denotation-assignment function of a formal model should be seen as reflecting the real association of meanings with expressions.

Continuing along these lines, then, it would appear that there is no direct relationship between the inferential component of the formal language and natural language – there being nothing in natural language that would directly correspond to inference. Inference, from this viewpoint, is merely the logicians' tool of getting a better grip on consequence, which itself is a matter of semantics (and can be defined via denotation-assignments). Inference is thus the theoretician's attempt to approximate consequence by finite and hence convenient means[5].

Needless to say, were this to be the sole way of aligning the formal model with a real language, then the inferentialist should best refrain from taking formal models seriously – for he does not believe that there is a meaning, independent of inference, that can be thus compared with the semantic interpretants of the formal language. But fortunately it is not the only way; there exists an alternative: we can say that where the model connects with reality is not semantics, but inference. We may then say that the model is adequate if the stipulated inferential rules explicate the inferential rules constitutive of the real language. And we may say that semantics is simply our way of seeing the inferential roles as distributed among individual expressions – as the expressions being mapped on their contributions to the inferential patterns they support.

This, of course, presupposes that inference, besides being a tool employed by the logician as an expedient within her pursuit of consequence, is also something already present within natural language. And this is precisely the basic inferentialist *credo*: that language is generally constituted by rules and that meaning, in particular, is constituted by inferential rules – so much so, that what makes an expression meaningful in the first place is the fact that it is governed by an inferential pattern.

To illustrate this, let me consider the simple case of classical logical constants. The denotationalist would say that classical logic can be seen as a model of natural language in so far as the logical constants denote the very same things as their natural language counterparts (*modulo* a reasonable simplification and idealization); and that the soundness and

---

fully-fledged expressions, on a par with constants, and hence if we base the languages of logic on a kind of grammar deviating from that underlying natural languages. (Regrettably, this essentially technical move has led some logicians to philosophical conclusions – such as that logic has discovered that natural languages contain variables in a covert way etc.)

[5] See Peregrin (2007).

completeness proof for the classical propositional calculus shows that the ensuing relation of consequence can be conveniently captured in terms of a couple of axioms and the rule MP. The inferentialist, by contrast, would say that classical logic models natural language in so far as the axiomatics (or the natural deduction rules) of classical logic capture the inferential patterns actually governing their counterparts in natural language (*modulo* a reasonable simplification and idealization); and that the soundness and completeness proof shows that the usual semantics can be seen as a way of presenting the inferential structure as distributed among individual expressions.

This brings us to the problem of the primitives, the 'unexplained explainers' the theory of language is to rest upon. The central concept of this enterprise is, no doubt, *meaning*. But when we look at early semantic theories within modern logic (what Tarski called "scientific semantics"), we see this concept fading into the background: the crucial concept appears to be that of truth. This might be explained by the fact that these theories never targeted the intuitive concept of meaning (except, perhaps, its 'extensional component'); however, an explicit defense of taking the concept of *meaning* as secondary to the concept of *truth* is also available: in its most elaborated version from Davidson. Thus, in these theories, the concept of truth is what generally stands as the 'unexplained explainer' of semantics – in the sense that this concept is either not explained at all, or is reduced to some non-semantic concepts and serves as the basis for the explanation of other semantic notions. In this order of explanation, *consequence* is construed as *truth-preservation* and inference as the best possible approximation to truth-preservation in terms of rules (besides this, there is the purely formalistic concept of inference, where it is stripped down to transformability according to an arbitrarily chosen set of rules, but this has little to do with semantics).

From the inferentialist perspective, however, the situation looks very different. From the perspective Brandom is advocating, the core of any linguistic practices is the *game of giving and asking for reasons*. And hence we should ask what conditions must be fulfilled by a language, understood as a collection of expressions, in order for it to be able to facilitate this game? It is clear that to *ask* for reasons, I need a way of *challenging* an utterance, i.e. I need some statements to count as a challenge to other statements. Thus, language must provide for statements that are *incompatible* with other statements. Contrastingly, to *give* reasons, I need a statement from which the statement constituting the utterance I strive to substantiate follows; and thus language must also provide for statements that are correctly inferable from other statements. Hence it seems that the key concepts upon which a Brandomian style theory of semantics must rest, and which it must take for granted, are those of *incompatibility* and *inference*.

Inferential semantics

A model based on the concept of inference consists primarily of a formal language L and a relation |-- between finite sequences of its statements and its statements. The structure of the model language approximates the grammatical structure of the natural language in question (the potentially infinite number of sentences of L must be certainly 'finitely generated', by means of a finite lexicon and a finite set of rules for producing expressions from expressions), and the relation approximates the inferential structure of the language (which the inferentialist sees as existing and as identifiable): hence $A_1, ..., A_n$ |-- A if the counterpart of A is correctly inferable from the counterparts of $A_1, ..., A_n$.

In a sense, the job of the inferentialist modeler is done once he produces this structure to his satisfaction. Of course, |-- must also be 'finitely generated', in this case resting on the grammatical structure of L, for there is no way of presenting an infinite relation other than to present some basic instances together with some recursive ways of extending them; and there is no nontrivial way of specifying such recursive ways save as resting on the recursive structure constitutive of its linguistic carrier, i.e. on the rules of grammar. Note, however, that there is no reason to assume that each expression will have an inferential pattern independent of those of other expressions (like '∧' does), nor that the patterns for more complex expressions will always be derivable from those for their parts. The only restriction is the 'finite generatedness'[6].

However, a traditionally-minded semanticist will probably not accept this kind of model as satisfactory. *Where is truth*?, *where is consequence*?, and indeed, *where is meaning*?, may well be asked. And, *why call a* purely syntactical *structure like the above one explication of language*?. Although I am convinced that to call this kind of structure "purely syntactical" is misguided[7], I think these questions do require attention; so it is reasonable for the inferentialist to develop the structure further.

First, can an inferentialist make sense of consequence as distinct from inference? I think that to some extent, she can. A paradigmatic example of such an enterprise can be found in Carnap's (1934) *Logical Syntax of Language*. What Carnap undertakes in the book is a purely inferentialist enterprise; but surprisingly, what he declares to be his ultimate aim is not to delimit the relation of *inference* (*Ableitbarkeit*), but the relation of *consequence* (*Folgerung*). And indeed he tries to define consequence – as distinct from inference – using his purely inferentialist means.

Now, having consequence, we can proceed to truth. For a traditional semanticist, consequence is in fact truth-preservation; and I see no reason for an inferentialist to disagree. The only thing he cannot agree to is to construe this as saying that the concept of consequence is *reducible* to that of truth. But if he wants to have consequence (or inference, if he thinks he can substantiate the denial of a gap between the two) as a concept more basic than truth, he can try to use the above relationship between truth and consequence as means of reducing the former concept to the latter one, rather than *vice versa.* He can say that truth is what is preserved by consequence[8].

---

[6] Cf. McCullagh (2003).

[7] See Peregrin (2006b).

[8] If this question is put into a form precise enough to be answered (which I tried to do in Peregrin, 2006a), it turns out that what a relation must fulfill are precisely the well-known Gentzenian structural

We are now approaching the last, and the most important, concept of the semantic battery - that of meaning. From the inferentialist viewpoint, meaning is usually said to be something like an inferential role[9], but as the concept of role is somewhat vague, we need an explication. I think that we can implicitly delimit the concept of inferential role by the following sets of constraints:

(a) Inferential roles uniquely determine inference; that is the inferential roles of A_1, ..., A_n and A uniquely determine whether $A_1, ..., A_n$ |-- A or not.

(b) The inferential role of a complex expression is uniquely determined by the inferential roles of their parts (this follows from the fact that inferential roles are identified with meanings and that the concept of meaning involves compositionality – as I have argued for elsewhere[10]).

(c) Inferential roles are a matter of nothing else than (a) and (b); that is, the inferential role is 'the simplest thing' that does justice to (a) and (b).

Do (a) – (c) answer the question of what an inferential role is? If we compare the concept of inferential role to that of number, it is *not* an answer on a par with von Neuman's saying that a number is the set of its predecessors (with zero being the empty set), but it *is* an answer in the sense of Quine's saying that a number is what is specified by the axioms of arithmetic[11]. And given that on closer inspection von Neuman's answer turns out to be not really an answer to the question *what is number?*, but rather one of many possible explications of the concept, I think that (a) - (c) do offer us a way of *answering* the question about the nature of inferential roles.

Of course, just as in the case of numbers, it makes sense to go on *explicating* inferential roles. How is this to be achieved? The most straightforward way is to provide a mapping of expressions on some kind of entities that would do justice to (a) – (c). Before doing this, we should probably prove that such a mapping exists at all – but this is easy. It is clear that there is a relation of intersubstitutivity *salva inferentiae* – let us write E ≈ E' iff for every inference A_1, ..., A_n |-- A iff A_1[E/E'], ..., A_n[E/E'] |-- A[E/E'] (where X[E/E'] is the result of replacing

---

rules, stating that for all sentences A, B and C and all finite sequences X, Y and Z of sentences it is the case that

$A \vdash A$

if $X,Y \vdash A$, then $X,B,Y \vdash A$

if $X,A,A,Y \vdash B$, then $X,A,Y \vdash B$

if $X,A,B,Y \vdash C$, then $X,B,A,Y \vdash C$

if $X,A,Y \vdash B$ and $Z \vdash A$, then $X,Z,Y \vdash B$

More precisely I assumed that what it takes, on the most general level, for a language *to have a semantics* is to classify its truth valuations into acceptable and unacceptable; and I have shown that an inference relation can be seen as effecting such a classification just in this case. The same holds, *mutatis mutandis*, for incompatibility.

[9] See Peregrin (2006b).

[10] See Peregrin (2005).

[11] See Quine (1969, p.45)

zero or more occurrences of E by E' in *X*). Now it is not difficult to see that mapping E on the class $[E]_{\approx}$ of all expressions that are equivalent with E does justice to (a)-(c).

Hence we can use the equivalence classes as explications of the inferential roles; but this does not lead to an illuminating explication. Let us call the *downstream inferential potential of a sentence* the set of all sentences from which the sentence follows and let us call the *upstream inferential potential* the set of all sentences which follow from it together with various collateral premises:

$$A^{\leftarrow} = \{S \mid S \mid\!\text{--}\; A\}$$
$$A^{\rightarrow} = \{\langle S_1, S_2, S\rangle \mid S_1,A,S_2 \mid\!\text{--}\; S\}$$

The *inferential potential* of *A* then can be seen as composed of these two parts.

$$A^{IP} = \{A^{\leftarrow},A^{\rightarrow}\}$$

An *inferential role* of an expression E can be now taken as a function mapping sentence contexts (appropriate for E) on inferential potentials, where a sentence context of a type appropriate for E is a sentence with one or more occurrences of an expression of the grammatical category of E replaced with three dots. (Thus, "... is an inferentialist" is a sentence context appropriate for "Bob", and "Bob ... an inferentialist" is one appropriate for "is".) The inferential role of E maps a sentence context on the inferential potential of the sentence which results form the substitution of E for the three dots of the context:

$$E^{IR}\,(SC(...)) = SC(E)^{IP}$$

Now the inferentialist's claim is that two expressions have the same inferential roles (in this sense) iff they are intersubstitutive salva inferantiae.

The obvious drawback is that, explicated in this way, the inferential roles do *not* comply with (a)-(c) above; indeed the inferential roles of any two different expressions come out as different. This is because the explicates are made up of linguistic expressions, distinguishable, as they are, one from another, whereas we now want to avoid distinguishing between inferentially equivalent ones. This poses a problem, though only one of a purely technical nature. Just as with all explications, to find the truly handy solution may take some ingenuity (for we seem unable to say in advance what exactly is meant by *handy*), but there are, I think, no deep philosophical problems involved.

Inference and incompatibility

We claimed that the other basic concept an inferentialist could use as an 'unexplained explainer' is incompatibility (or incoherence[12]). Can we develop our inferential model of language to accommodate it? If we were to accept that incompatibility is *reducible* to inference, then there is one obvious way, a very traditional one: to say that two sets of sentences are *incompatible* if every sentence is inferable from their union.

In this way, incompatibility becomes dependent on the expressive richness of the language in question. Something, it would seem, might come out as incoherent only because the language lacks, by pure chance, all sentences that would not be inferable from it. And though I think that in the case of a natural language we are entitled to presuppose some kind of 'expressive saturatedness', this feature of the reduction of incompatibility to inference makes it a little bit suspicious.

There is also a way of making room for the concept of incompatibility by generalizing the concept of inference: by saying that inference is not a relation between finite sequences of statements and statements, but rather that it is a relation between finite sequences of statements and finite sequences of statements *of length zero or one*. The extension is natural in that the resulting relation would comply with the Gentzenian structural rules and can be seen as half-way to the multi-conclusion version of inference as we know it from the sequent calculus.

The way chosen by Brandom consists in taking incompatibility as *the* 'unexplained explainer', explicating inference as compatibility-preservation: claiming $A_1, ..., A_n \vdash A$ is construed as claiming that whatever is incompatible with A is incompatible with $A_1, ..., A_n$, i.e. that whatever is compatible with the former is compatible with the latter. (This indicates that there is a sense in which compatibility assumes the role of truth in his system; we will return to this later.)

Is there a difference between basing the formal semantics on inference and basing it on incompatibility? We have seen that the concepts may be thought of as interdefinable; but the interdefinability is not unproblematic. Starting from inference we can take incompatibility in stride, we saw, only with some provisos. Conversely, basing inference on incompatibility might seem more straightforward; but it contains a notable snag. The reduction of inference to incompatibility contains quantification over all sentences, hence the inference relation based on a finitary incompatibility relation might not be itself finitary.

This may be more important than first meets the eye. Recall that when we considered the general structure of a formal language we distinguished between axiomatics (proof theory) and semantics (model theory) and we held for important which one of them is the 'point of contact' between the model and a real language. Now what is the crucial difference between these two components? After all, both of them aim at a specification of a relation among sentences (inference resp. consequence), or a set of sentences (theorems resp. tautologies) and

[12] We follow Brandom's terminology, according to which incoherence is simply self-incompatibility (and incompatibility is the incoherence of union).

we know that at least in some cases (classical propositional logic) even their outcome is the very same.

The basic difference, I think, is that while the proof-theoretic enterprise is principally a matter of finite generation, there is no such restriction for the model-theoretic specification. As a result, this specification may happen to be finitary (as in the case of classical propositional logic), but in general need not. Now the point is that if we have a finitely generated relation of incompatibility and use it to define the relation of inference, the resulting definition *may no longer be finitary*. This means that a proof-theoretical delimitation of incompatibility may yield a definition of inference that is, by its nature, more model-theoretic than proof-theoretic. In this sense, starting from incompatibility may lead us, in a way that is easy to miss, into the realm of formal semantics.

It is also worth noticing that this definition of inference leads us, almost inescapably, into the realm of *classical* logic. Defining inference as containment of a set associated with the conclusion in the intersection of sets associated with the premises amounts (independently of whether the sets are sets of incompatible sets of sentences or of anything else) to giving the resulting logic a *Boolean* semantics - i.e. to having it semantically interpreted in a Boolean (rather than, say, Heyting) algebra. This interpretation, then, leaves us almost no other possibility than to define the logical operators in the Boolean, i.e. classical way. I find this problematic, because, as I argued elsewhere (see Peregrin, to appear), I find intuitionist logic more natural than classical from the inferentialist viewpoint.

We have said that having an inferential model of language, the reason for going on to develop a formal semantics may be the desire to explicate the concept of meaning (which we may or may not submit to); but now we see that we might be driven into semantics in a sense against our will, by deciding to start from incompatibility rather than from inference.

Incompatibility semantics and possible world semantics

Now we are finally homing in on our initial question (3). Does basing a semantic model on the concept of incompatibility, rather than on inference, or indeed on a more traditional concept, like that of a possible world, give us any advantage? Clearly, by granting ourselves the concept of incoherence, and hence coherence, we grant ourselves all we need to introduce the concept of possible world. It is well known that a possible world can be seen as a maximal coherent set of sentences; for the sake of perspicuity, let us distinguish, as is sometimes done, between a possible world as such, and a *possible world story*, the corresponding set of sentences (true w.r.t. the possible world). What we can take to be a possible world story is any set of sentences that is not incoherent, but any of its proper superclass is incoherent. The problem with turning incompatibility semantics into a possible world semantics then seems to consist in the fact that we do not have the concept of truth, hence we cannot talk about a sentence being true w.r.t. a possible world.

This problem, however, is easily overcome. It is enough to realize that a possible world is complete in the sense that it leaves no possibilities open. Either something is the case, or it is not; there is no room for something being open in the sense that something could be coherently added. Hence to be true w.r.t. a possible world is to be compatible with the corresponding world story, i.e. to be part of the world-story. (This explains the hint we made above – namely that within the Brandomian setting, there is a sense in which compatibility assumes the role of truth.) This means that if we call any maximal coherent set of sentences a possible world story and if we say that a sentence is true w.r.t. the corresponding possible world iff it is compatible with the set, we have a recasting of the incompatibility semantics into a possible world semantics.

Given what Brandom calls an incompatibility frame, i.e. a set (of sentences) and a subset of its powerset (of incoherent theories) closed to supersets, let us denote the set of all maximal coherent sets of sentences W and let w range over its members. Let us write $w \Vdash p$ for p is true in w (which, as we already know, amounts to $p \in w$). Note also that $w \Vdash p$ iff $w \models p$, i.e. iff w incompatibility entails p. ($w \models p$ iff every q incompatible with p is incompatible with w, i.e. iff every q compatible with w is compatible with p. i.e. iff every element of w is compatible with p i.e. iff p is compatible with w i.e. iff p is true in w.) Hence p is true in w iff it is entailed by w iff it is compatible with w iff it is an element of w.

This also gives us all the resources needed to define the simplest (S5-) kind of necessity: as we have possible worlds, we can say that p is necessary iff it is true w.r.t. all the possible worlds and that it is possible iff it is true w.r.t. at least one of them. To introduce any other kind of necessity, we would need, moreover, something amounting to the Kripkean accessibility relation among worlds; and it is important to realize that we have *nothing* of this kind. The point is that the accessibility relation spells out as a kind of 'second-level compatibility': though any two *worlds* are incompatible (in the sense that the union of their stories is incoherent) some worlds are 'less incompatible than others' (for example, by sharing the same physical laws etc.[13]).

In view of this, it seems puzzling that Brandom introduces a notion of necessity that appears to be more complicated than the simplest one. His definition of the necessity operator is the following:

$$X \cup \{Lp\} \in \text{Inc iff } X \in \text{Inc or } \exists Y[X \cup Y \notin \text{Inc and } Y \mid\neq \{p\}]$$

This says, in effect, that p is necessary w.r.t. a world iff p is not entailed by anything that is compatible with this world[14]:

[13] Of course, in cases in which the accessibility relation is not symmetric it may be awkward to see it as expressive of 'compatibility', but we leave this aside.

[14] In this way we have made a long story short. To tell the long story explicitly, we would have to say that to make the incompatibility semantics into a regular Kripkean one, we need an accessibility relation. That is, we need a relation such that Lp is true w.r.t. a possible world w just in case p is true w.r.t. all worlds accessible from w. The usual way to extract the accessibility relation from the world stories is to say that the world w' will be accessible from the world w iff whatever is necessarily true in the latter, is true in the former, i.e.

$$wRw' \text{ iff } \{p \mid Lp \in w\} \subseteq w'$$

(*) w ||= L*p* iff ∀Y(if w ||= Y, then Y |= *p*)

But it is not difficult to show that this unperspicuous definition reduces to a very simple one:

(**) w ||= L*p* iff |= *p*

(To get from the right-hand side of (*) to that of (**) it is enough to instantiate Y as the empty set. To get, *vice versa*, from (**) to (*), it is enough to realize that for every Y, |= *p* entails Y |= *p*.)[15]

This means that basing semantics exclusively on the pure, 'first-level' concept of compatibility gives us resources to make sense of merely the most common-or-garden variety version of necessity. (See Appendix for an illustration of how an additional level can elicit different logic.) In contrast to this, it seems that the logical vocabulary of natural language is far richer and diverse. We have various kinds of necessity words – words explicitating varieties of material inferences from causal necessity or epistemic necessity to something very close to the S5-kind of logical necessity.

A response to this might be *But this is as it should be – perhaps Brandom's is the only* purely logical *version of necessity – others being contaminated, in such or another way, by materialness.* It might be argued that other versions of necessity correspond to such things as physical necessity that are surely not purely logical matters. But I do not think this answer, though partly true, should be fully embraced.

It is obvious that a natural language never comes with a border neatly separating its logical vocabulary from the rest of its words. (Hence just as it is futile, as Quine taught us, to look for a seam between the analytic and the synthetic, it is equally futile to look for one between the logical and the material – for natural language is generally *seamless*.) Moreover, I do not believe that this should be seen as a shortcoming and make us search for the purely logical in ways bypassing natural language altogether[16]. I think that the reason behind the apparent

---

This definition guarantees that a sentence necessarily true in a possible world will be true in all worlds accessible from it; but if it be the accessibility relation underlying our current necessities, we have to show also the converse, namely that a sentence is necessarily true in a possible world if it is true in all worlds accessible from it. Hence we must show that

If $p \in w'$ for every w' such that wRw', then $Lp \in w$

But given Brandom's definition, this is easy; and it is also easy to show that R is an equivalence.

[15] Pleitz et al. (to appear) have attempted to modify the definition so as to yield a less trivial modal logic, which they achieved by restricting the range of Y in (*) to singletons. This indeed yields them a logic weaker than S5. But the restriction they employed seems to me to be so unmotivated that I cannot see this result as of any other than purely technical interest.

[16] In general, I think we should shy away from diagnosing natural language as 'imperfect'. The millennia of natural selection responsible for its current shape are more likely to have streamlined and perfected it – though perfected it *in its own way*. It is probable that the imperfection diagnosis (a matter of course for many classical analytic philosophers, and the driving force behind much of their

inextricability of the logical from the extralogical, just like that of the analytic from the synthetic, is that these distinctions are of our own making – that they are in the eye of the beholder rather than in language itself.

To avoid any misunderstanding: there is no doubt that these distinctions are extremely useful and they help us understand the functioning of natural language. But we must not forget that they are a matter of the Bohr-atom kind of idealization – a kind of idealization that is, beyond doubt, an important (if not indispensable) means of our understanding things like language or the structure of matter, but an idealization nevertheless. To see the idealized model as an exact copy of the thing modeled and to believe that whatever is displayed by the former, but does not seem to be displayed by the latter, must therefore be somehow 'hidden under its surface', is to misunderstand the whole enterprise of model-aided understanding.

What are the implications of all this for the question of whether there is *the* set of logical operators and hence *the* logic? I do not think the answer is in the positive. Apart from there being some leeway in *how* we explicitate[17] inference, there is the question what inference exactly is. Most usually it is taken as a relation between finite sequences (or finite sets) of statements and statements; but after Gentzen's (1934) seminal work it is not unusual to take it alternatively as a relation between finite sequences (or finite sets) of statements and finite sequences (or finite sets) of statements; and we may also consider some possibilities between the two extremes. Besides this, it is usual to assume that inference is to comply with the Gentzenian, structural rules; but even this assumption is sometimes alleviated (witness the research done in the field of the so called *substructural logics*[18]).

It turns out that when we take inference to be single-conclusion and complying with the structural rules, the most natural tools for its explicitation are the intuitionist operators; whereas when we allow for multiple conclusions, what we gain will be classical logic (see Peregrin, to appear, for details). And as I am convinced that there are good reasons to believe that it is the single-conclusion variety that is more basic, I think that it is intuitionist logic that is the 'most basic' logic from the viewpoint of an inferentialist.

The reason why I think that the single-conclusion inference is more basic is that inference may exist (according to the Brandomian picture to which I fully subscribe in this respect) only in the form of the inheritance of certain social statuses, prototypically commitments. It may exist through the fact that members of a community hold a person to be committed to something in virtue of her being committed to something else – even prior to their being able to *say* this. (This is essential, for it is only the need for making this explicitly sayable that is the *raison d'être* of the existence of logical particles and other expressive resources of natural language that do make it sayable.) And while it is clear what it could take practically to hold

---

philosophy) results from language being expected to fulfill purposes different from its intrinsic ones – i.e. those for which it was selected.

[17] I use the verb *explicitate* in the Brandomian sense of *making explicit*, whereas I am reserving the verb *explicate* for use in the Carnapo-Quinean sense of devising an exactly delimited substitute for a vague concept.

[18] See Restall (2000).

someone to be committed to doing something (above all, but not only, to be prepared to sanction him, when he does not do it), and hence also what it is to hold someone to be committed to doing something in virtue of being committed to doing other things, it is far from so clear what would it mean to hold somebody committed to doing one of a couple of things – or at least this would not seem to be a candidate for a perspicuous communal practice underlying, rather than brought about by, the occurrence of the explicit 'or' and other logical particles[19].

Logical vocabulary of natural language vs. logical vocabulary of formal languages

I take it that Brandom's stance is that a force behind the development of natural language is the tendency of its users to 'make it explicit' – i.e. to introduce means which would allow them to formulate, in the form of explicit claims, what was only implicit in their practices before. Hence if they endorse the inferences such as from *This is a dog* to *This is an animal*, or from *Lightning now* to *Thunder shortly*, they are to be expected to develop something of the kind of *if ... then* to be able to say *If this is a dog, then this is an animal* or *If lightning now, then thunder shortly* (and later perhaps more sophisticated means which would allow them to further articulate such claims as *Every dog is an animal* or *Lightning is always followed by thunder*). I take it that this is the point of logical vocabulary of a natural language like English.

Now, besides natural languages we have formal languages with their logical constants, such as the language of Brandom's incompatibility semantics with its **N**, **K** and **L**. What is the relationship between such expressions and the logical vocabulary of natural language? I must say I am not sure what Brandom's answer is supposed to be.

There seem to be two possibilities. On the one hand, there is the view that what logic is after is something essentially non-linguistic, some very abstract structure which can be identified and mapped out without studying any real language (be it a structure of a hierarchy of functions over truth values, individuals and possible worlds, or a general structure of incompatibility). This is a notion of logic, in Wittgenstein's (1956, §I.8) words, as a kind of "ultraphysics". In this conception, any possible language is restricted by the structure discovered by logic (for it is only within the space delimited by the structure that a language can exist), and hence there is a sense in which philosophy of language is answerable to logic.

---

[19] It is important to realize that though this may seem unproblematic in some specific cases (for example to hold a person *A*, who accepted a thing from a person *B*, to be committed to *either* giving *B* another thing in exchange, *or* returning *B* the original thing, does not seem to be any more problematic than, and indeed no different from, holding *A* to be committed to giving *B* a thing), multiple-conclusion inference would require a wholly *general* notion of the disjunctive combination of commitments. And it is very difficult to imagine what it would take practically to hold a person to be committed to doing one of an assortment of unrelated things, without the means for stating the commitment explicitly.

On the other hand, we may hold that the primary subject matter of logic is the logical means (words and constructions) of our real languages (especially those, that seem to be identifiable across languages and which something must possess in order to qualify as an element of the extension of our term "language"). It is, then, only derivatively that logic studies certain abstract structures – they are structures distilled out of natural language similarly to how the Bohr atom model is distilled out of our empirical knowledge of the inside of the atom. (Of course, this does not make the study of such structures unimportant – this kind of 'mathematization' has become the hallmark of advanced science.) Hence there is a sense in which logic is answerable to the philosophy of language rather then vice versa.

Given these two options, I vote for the latter; for I am strongly convinced that the former one is not viable. In my opinion, formal languages of logic are not means of direct capturing of abstract logical structures, but rather *simplified models* of natural language – as I have already pointed out, they provide for Wittgensteinian *perspicuous representation*. They abstract from many features of natural languages, idealize the languages in many ways, but let us see something as its *backbone* quite clearly. Their logical vocabulary *explicates* that of natural language and though there may be some feedback, it is not in competition with it.

I do not believe that any artificially created logical symbol, be it the traditional '⊃' or Brandom's '**K**', ever gets employed by the speakers of English in such a way that it can viably *compete* with *if ... then* or *and*. True, logical theories based on such symbols have helped us see the workings of our natural logical vocabulary in a much clearer light, and in some exceptional cases, as in the texts of some eccentrically meticulous mathematicians (such as Giuseppe Peano or Gottlob Frege), can even assume their role; but I do not believe that they can be generally seen as viable alternatives, or even successors, to the natural logical vocabulary.

Now Brandom does not seem to concur with me on this point. He tries to extract the logical operators directly from the structural features of incompatibility. This would indicate that he goes for the option I reject. But the truth, probably, is that his understanding of his operators coincides with neither of my two options. If this is true, then I would be eager to learn what his option is.

Conclusion

To sum up, I think that formal semantics may be a very illuminating enterprise; and this is the case even when we subscribe to pragmatism and to the inferentialist construal of the nature language. Of course, if you are a pragmatist and an inferentialist, you must understand the enterprise of formal semantics accordingly – not as an empirical study of the ways in which words hook on things, but rather as a way of building a 'structural' model of language explicative of its semantic (and hence eventually inferential) structure.

From this viewpoint, excursions into the realm of formal semantics as exemplified by Brandom in his Lecture V, seem to me very useful. However, I think we need a deeper explanation of why they are useful and what they can teach us. I believe not only that the logical analysis of natural languages without the employment of logical structures is blind, but also that the study of logical structures without paying due attention to the way they are rooted within natural languages is empty.

APPENDIX

Let me first summarize the basic conceptual machinery of Brandom and Aker[20] (however, I will deviate from their terminology wherever I find it helpful; and I will also use the more traditional signs for logical operators). Given a language L (understood as a set of sentences), a set Inc $\subseteq$ Pow(L) is called an *incoherence property* iff for every finite $X,Y \subseteq L$, if $X \subseteq Y$ and $X \in$ Inc, then $Y \in$ Inc. <L,Inc> will be called a *standard incoherence model*[21]. We write I(X) as a shorthand for

$\{Y \mid X \cup Y \in \text{Inc}\}$

and we write $X \models p$ as a shorthand for

$\forall Y$: if $Y \cup \{p\} \in$ Inc, then $Y \cup X \in$ Inc.

Given L is the set of sentences of the modal propositional calculus (based on $\wedge$, $\neg$ and $\Box$), Inc is, moreover, supposed to fulfill the following postulates

($\neg$) $X \cup \{\neg p\} \in$ Inc iff $X \models p$.

($\wedge$) $X \cup \{p \wedge q\} \in$ Inc iff $X \cup \{p,q\} \in$ Inc.

($\Box$) $X \cup \{\Box p\} \in$ Inc iff $X \in$ Inc or there is an Y such that $X \cup Y \notin$ Inc and $Y \not\models p$.

Possible worlds and intensions as based on incoherence

Given an incoherence property Inc, we define a set PW_{Inc} (of 'possible worlds') and, for every class X of statements, its intension $\|X\|_{\text{Inc}}$ (the class of all those possible worlds w.r.t. which it is true).

Definition.

$\text{PW}_{\text{Inc}} \equiv_{\text{Def.}} \{X \mid X \notin \text{Inc}$ and there is no $Y \notin \text{Inc}$ so that $X \subsetneq Y\}$ (possible worlds)

$\|X\|_{\text{Inc}} \equiv_{\text{Def.}} \{Y \mid Y \in \text{PW}_{\text{Inc}}$ and $X \cup Y \notin \text{Inc}\}$ (the intension of X)

(The index will be left out wherever no confusion will be likely.) We will use the letters w, w', w'', ... to range over those sets of statements that are possible worlds. Now we prove some auxiliary facts about possible worlds and intensions.

[20] I reflect the fact that the Appendix is the common work of the two authors (hereafter B&A).

[21] I use the term *model* instead of B&A's *frame*, for, as we will see later, the natural counterparts of these structures within Kripkean semantics are models rather than frames. (As incompatibility semantics is built directly from language, there is nothing that would correspond to the concept of *frame* of standard Kripkean semantics - i.e. the concept of a space of a possible world with the relation of accessibility, taken in isolation of any language) Besides this, I base the definition directly on the incoherence property (rather than on the incompatibility function derived from it as B&A do).

Lemma.

1. $w \in \|X\|$ iff $X \subseteq w$
2. $X \notin$ Inc iff there is an w so that $X \subseteq w$, i.e. so that $w \in \|X\|$
3. $X \in$ Inc iff there is no w so that $X \subseteq w$, i.e. $\|X\| = \emptyset$
4. $\|X \cup Y\| = \|X\| \cap \|Y\|$
5. if $I(Y) \subseteq I(X)$, then $\|X\| \subseteq \|Y\|$
6. if $I(Y) \not\subseteq I(X)$, then $\|X\| \not\subseteq \|Y\|$

Proof.

1. According to (1), $w \in \|X\|$ iff $X \cup w \notin$ Inc, which, given that nothing that is not a part of w is not compatible with it, is the case iff $X \cup w \subseteq w$ and henced iff $X \subseteq w$.

2., 3. obvious

4. $w \in \|X \cup Y\|$ iff $X \cup Y \subseteq w$, that is iff $X \subseteq w$ and $Y \subseteq w$, and hence iff $w \in \|X\|$ and $w \in \|Y\|$.

5. Suppose $I(Y) \subseteq I(X)$. This is the case iff $\overline{\mathbf{I}(X)} \subseteq \overline{\mathbf{I}(Y)}$, and hence only if $\overline{\mathbf{I}(X)} \cap PW \subseteq \overline{\mathbf{I}(Y)} \cap PW$, i.e. only if $\|X\| \subseteq \|Y\|$.

6. Suppose that $I(Y) \not\subseteq I(X)$, i.e. that there is a Z such that $Z \in I(X)$ and $Z \notin I(Y)$. In other words, $Z \cup X \in$ Inc and $Z \cup Y \notin$ Inc, which, according to 2. and 3., is the case iff there is a w such that $w \in \|Z \cup X\|$ and there is no w such that $w \in \|Z \cup Y\|$. This implies that there is a w such that $w \in \|Z \cup X\|$ and $w \notin \|Z \cup Y\|$. This in turn implies that $w \in \|Z\|$ and $w \in \|X\|$ and ($w \notin \|Z\|$ or $w \notin \|Y\|$) and hence that $w \in \|X\|$ and $w \notin \|Y\|$.

Having proved this, we can prove that the assignment of intensions is isomorphic to the assignment of incompatibility sets; hence that the possible worlds semantics based on PW and $\|...\|$ is a faithful representation of the underlying incompatibility semantics.

Theorem.

$I(X) = I(Y)$ iff $\|X\| = \|Y\|$

Proof. The direct implication is trivial, as $\|X\| \subseteq \overline{\mathbf{I}(X)}$. The indirect one follows from the last two clauses of the previous lemma. This shows that as in the case of ordinary modal logics, the semantic value of a sentence can be identified with the set of all those possible worlds in which it is true.

Incompatibility semantics and the modal logic S5

Let us call the logic to which this incompatibility semantics gives rise (i.e. the resulting relation of consequence) *standard incompatibility logic* (*SIL*), and let us investigate its relationship to the standard systems of modal logic. (B&A have already proved that SIL is equivalent to S5, but I present a proof which is more suitable for what will follow.)

Lemma. Given (□), it is the case that

$X \cup \{\Box p\} \in$ Inc iff $X \in$ Inc or $\not\models p$.

Proof. It is obviously enough to prove that provided $X \notin$ Inc, $\not\models p$ iff there exists an Y such that $X \cup Y \notin$ Inc and $Y \not\models p$. The indirect implication follows from instantiating Y as the empty set; the direct one follows from the obvious fact that if $\not\models p$, then $Y \not\models p$ for any Y.

Theorem. $w \in ||\Box p||$ iff $\forall w\, w \in ||p||$; hence

$||\Box p|| = \{w \mid$ for every v: if wRv, then $v \in ||p||$), where R = PW×PW

Proof. According to the theorem just proved, $w \cup \{\Box p\} \in$ Inc iff $w \in$ Inc or $\not\models p$, hence iff $\not\models p$. Moreover, $w \cup \{\Box p\} \in$ Inc iff $w \notin ||\Box p||$; and $\not\models p$ iff p is incompatible only with incoherent sets, which means that it is compatible with every possible world and hence that $w \in ||p||$ for every w.

Corollary. Let <*L*,Inc> be a standard incompatibility frame.incoherence model. Then there exists an equivalent Kripkean model <W,R,V> with R = W×W, i.e. there exists a (one-one) mapping i of PW_{Inc} on W such that for every $p \in L$: $p \in w$ iff $V(p,i(w))=1$.

Proof. In view of the theorem we can simply take W = PW_{Inc}, i as the identity mapping, R = $PW_{Inc} \times PW_{Inc}$ and $V(p,i(w)) = 1$ iff $p \in w$.

Corollary. S5-validity entails SIL-validity.

Theorem. Let <W,R,V> be a Kripkean model with R=W×W. (We will use the variables v, v', v'', ... to range over elements of W.) Define

Inc = $\{X \mid$ there is no v such that $V(p,v)=1$ for every $p \in X\}$

Then Inc is an incoherence property and there exists a (not necessarily one-one) mapping i of W on PW such that $V(p,v) = 1$ iff $p \in i(v)$.

Proof. Let us call X *impossible* iff there is no $v \in W$ such that $V(p,v)=1$ for every $p \in X$ (thus, Inc is the set of all impossible sets). Let us write $V(X,v)=1$ a shorthand for $V(p,v)=1$ for every $p \in X$ .That Inc is an incoherence property is obvious. It is also obvious that for every v there will be an element $i(v) = \{p \mid V(p,v) = 1\}$ of PW such that $V(p,v) = 1$ iff $p \in i(v)$. Thus the only thing that must be shown is that Inc fulfills (¬), (∧) and (□). If we rewrite the postulates according to our definition of Inc, we get

(∧*) $X \cup \{p \wedge q\}$ is impossible iff $X \cup \{p,q\}$ is impossible

(¬*) $X \cup \{\neg p\}$ is impossible iff

$\forall Y$(if $Y \cup \{p\}$ is impossible, then $Y \cup X$ is impossible)

(□*) $X \cup \{\Box p\}$ is impossible iff X is impossible or

$\exists Y(Y \cup \{p\}$ is impossible, whereas Y is possible)

Whereas (∧*) is obvious, (¬*) and (3) are slightly more involved.

Let us start with (¬*): To prove the direct implication, assume that $Y \cup \{p\}$ is impossible, i.e that $V(p,v)=1$ for no v such that $V(Y,v)=1$. It follows that $V(\neg p,v)=1$ for every v such that

V(Y,v)=1, i.e. that V(X,v)=1 for no v such that V($Y\in$W,v)=1, and hence that V($Y\cup X,v$)=1 for no v. To prove the indirect one it is enough to instantiate Y as $\neg p$.
As for (□*): To prove the direct implication, observe that if V(X,v)=1 for some v, then V(□p,v)=0, and hence that there must be a v' such that V(p,v')=0. But then, $i(v')\cup\{p\}$ is impossible, whereas $i(v')$, being the set of all sentences true in v', is possible. To prove the indirect implication, observe that if V(X,v)=1 for some v, there must be a v' such that V(p,v')=0; hence V(□p,v'')=0 for every v''; and hence $X\cup\{$□$p\}$ is impossible.

Corollary. SIL-validity entails S5-validity (and hence they coincide).

An extended incompatibility semantics and the modal logic B

Now we indicate how the addition of one more level of incompatibility may lead to a logic different from S5.

Definition. An *extended incoherence model* is an ordered triple <L,Inc,QInc>, where <L,Inc> is a standard incoherence model and QInc$\subseteq$Pow(Pow(L)) such that:

(a) if $U\in$QInc and $U\subseteq U'$, then $U'\in$QInc

(b) if $\cup U\notin$Inc, then $U\notin$QInc

(c) $\{X_i\}_{i\in I}\notin$QInc iff there are possible worlds $\{w_i\}_{i\in I}$ so that $X_i\subseteq w_i$ for every $i\in I$ and $\{w_i\}_{i\in I}\notin$QInc

If $\{X_i\}_{i\in I}\notin$QInc, then the sets $\{X_i\}_{i\in I}$ are called *quasicompatible*.

(The intuitive sense behind this definition is the following: (a) simply states that QInc is an incoherence property; (b) states that it is a weakening of Inc in the sense that every two compatible sets are quasicompatible; and (c) states that to be quasicompatible is to be parts of quasicompatible possible worlds. It is obviously this last clause that makes QInc suitable for emulating a Kripkean accessibility relation.)

Definition. Let <L,Inc,QInc> be an extended incompatibility model. We say that a possible world w' from PW_{Inc} is *accessible from* the possible world w', $wR_{Inc}w'$, iff $\{w, w'\}\notin$QInc.

Lemma. The accessibility relation is reflexive and symmetric; i.e. every world is accessible from itself and every world is accessible from every world that is accessible from it.
Proof. Obvious.

Definition. We replace the above (□) by

(□') $X\cup$□$p\notin$Inc iff for every Y: if <$X\cup$□p,Y>$\notin$QInc, then $Y\cup p\notin$Inc.

The modal logic based on this kind of incompatibility semantics will be called *extended incompatibility logic* (*EIL*).

Theorem. $w \in ||\Box p||$ iff for every w' such that wRw' it is the case that $w' \in ||p||$

Proof. It is clear that

$w \cup \Box p \notin$ Inc iff for every Y: if $\langle w \cup \Box p, Y \rangle \notin$ QInc, then $Y \cup p \notin$ Inc,

and as $w \cup \Box p \notin$ Inc iff $w \cup \Box p = w$, that

$w \cup \Box p \notin$ Inc iff for every Y: if $\{w,Y\} \notin$ QInc, then $Y \cup p \notin$ Inc.

This can be rewritten as

$w \in ||\Box p||$ iff for every Y such that $\{w,Y\} \notin$ QInc there is a w' such that $w' \in ||Y \cup p||$.

Hence what we must prove is that

(*) for every w' such that wRw' it is the case that $w' \in ||p||$

is equivalent to

(**) for every Y such that $\{w,Y\} \notin$ QInc, there is a w'' such that $w'' \in ||Y \cup p||$.

That (*) follows from (**) is obvious: restricting the Y's of (**) to possible worlds yields us

for every w' such that $\{w,w'\} \notin$ QInc, there is a w'' such that $w'' \in ||w' \cup p||$,

where $\{w,w'\} \notin$ QInc is the sane as wRw' and $w'' \in ||w \cup p||$ iff $w''=w'$ and $w'' \in ||p||$ (as $||w' \cup p|| \subseteq ||w'|| \cap ||p||$, $w'' \in ||w' \cup p||$ only if $w'' \in ||w'||$, which is possible only if $w''=w'$, and $w'' \in ||p||$).

Let us prove that, conversely, (**) follows from (*). Hence assume (*) and assume that $\{w,Y\} \notin$ QInc. According to (c) of the definition of the extended incoherence model, there is a w' such that $Y \subseteq w'$ and $\{w,w'\} \notin$ QInc. In other words, there is a w' such that wRw' and $Y \subseteq w'$. According to (*), $\{p\} \subseteq w'$; hence $Y \cup \{p\} \subseteq w'$, i.e. $w' \in ||Y \cup p||$.

Corollary. Let $\langle L,\text{Inc},\text{QInc} \rangle$ be an extended incmpatibility model. Then there exists an equivalent Kripkean model $\langle W,R,V \rangle$ with R reflexive and symmetric, i.e. there exists a mapping i of PW_{Inc} on W such that for every $p \in L$ and every $w \in PW$ it is the case that $p \in w$ iff $V(p,i(w))=1$.

Proof. In view of the previous theorem we can simply take $W = PW_{Inc}$, i as the identity mapping, $R = \{\langle w,w' \rangle \mid \langle w,w' \rangle \notin \text{QInc}\}$ and $V(p,i(w)) = 1$ iff $p \in w$.

Corollary. S4-validity entails EIS-validity.

Theorem. Let $\langle W,R,V \rangle$ be a Kripkean model for a language L with R reflexive and symmetric. Define

$\text{Inc} = \{X \mid \text{there is no } v \text{ such that } V(X,v) = 1\}$

$\text{QInc} = \overline{\{U \mid \cup U \notin \text{Inc}\} \cup \{\langle X,Y \rangle \mid V(X,v) = 1, V(Y,v') = 1 \text{ and } vRv' \text{ for some } v \text{ and } v'\}}$

Then $\langle L,\text{Inc},\text{QInc} \rangle$ is an extended incoherence model and there exists a (not necessarily one-one) mapping i of W on PW such that $V(p,v) = 1$ iff $p \in i(w)$.

Proof. That Inc is an incoherence property is obvious. Let us check that QInc is a quasiincoherence property. The definition says that the sequence of sets is quasicoherent iff either the union of the sets is coherent, or it consists of a pair of accessible possible worlds. In both the cases (a) is obviously fulfilled. Also due to the first part of the definition, (b) is fulfilled. And also (c) is obviously fulfilled in both cases. So what is left to be proved is that the postulates for logical connectives are fulfilled, and of those ($\wedge$) and ($\neg$) are the same as before. So we are left with ($\Box$'):

(*) $X\cup\Box p\notin \mathrm{Inc}$ iff for every Y: if $\langle X\cup\Box p,Y\rangle\notin \mathrm{QInc}$, then $Y\cup p\notin \mathrm{Inc}$.

What we know is that given w and w' range over W,

$V(\Box p,w) = 1$ iff for every w' such that wRw' it is the case that $V(p,w') = 1$.

hence

(**) $w\cup\Box p\notin \mathrm{Inc}$ iff for every w such that if $\langle w,w'\rangle\notin \mathrm{QInc}$ it is the case that $w'\cup p\notin \mathrm{Inc}$.

Thus we must prove (*), given (**). Let us start with the direct implication: assume $X\cup\Box p\notin \mathrm{Inc}$ and $\langle X\cup\Box p,Y\rangle\notin \mathrm{QInc}$ and head for proving $Y\cup p\notin \mathrm{Inc}$. $\langle X\cup\Box p,Y\rangle\notin \mathrm{QInc}$ entails that there are w' and w'' such that $X\cup\Box p\subseteq w$, $Y\subseteq w'$ and $\langle w,w'\rangle \notin \mathrm{QInc}$. However, $X\cup\Box p\subseteq w$ entails that $w\cup\Box p\notin \mathrm{Inc}$, and hence, according to (**), that for every w'' such that $\langle w,w''\rangle\notin \mathrm{QInc}$ it is the case that $w''\cup p\notin \mathrm{Inc}$. Hence, as $\langle w,w'\rangle \notin \mathrm{QInc}$, it is the case that $w'\cup p\notin \mathrm{Inc}$, and as $Y\subseteq w'$, that $Y\cup p\notin \mathrm{Inc}$. To prove the inditect implication, assume that for every Y, if $\langle X\cup\Box p,Y\rangle\notin \mathrm{QInc}$, then $Y\cup p\notin \mathrm{Inc}$; and hence especially that for every w, if $\langle X\cup\Box p,w\rangle\notin \mathrm{QInc}$, then $w\cup p\notin \mathrm{Inc}$. As $\langle X\cup\Box p,w\rangle\notin \mathrm{QInc}$ iff $\langle w',w\rangle$ for some w' such that $X\cup\Box p\subseteq w'$, hence $X\cup\Box p\notin \mathrm{Inc}$.

References

Brandom, B. : *Between Saying and Doing: Towards an Analytic Pragmatism*.

Carnap, R. (1934): *Logische Syntax der Sprache*, Springer, Vienna.

Davidson, D. (1986): 'A Nice Derangement of Epitaphs', LePore, E. (ed.): *Truth and Interpretation: Perspectives on the Philosophy of Donald Davidson*, Blackwell, Oxford, 433-446.

Gentzen, G. (1934): 'Untersuchungen über das logische Schliessen I-II', *Mathematische Zeitschrift* 39, 1934, 176-210.

Haugeland, J. (1978): 'The nature and plausibility of cognitivism', *Behavioral and Brain Sciences* 1, 1978, 215-260.

Lewis, D. (1972): 'General Semantics', Davidson, D. & Harman, G. (eds.): *Semantics of Natural Language*, Reidel, Dordrecht, 1972, 169-218.

Lewis, D. (1975): 'Languages and Language', K. Gunderstone (ed.): *Minnesota Studies in the Philosophy of Science VII*, University. of Minnesota Press, Minneapolis.

McCullagh, M. (2003): 'Do inferential roles compose?', *Dialectica* 57, 431-438.

Montague, R. (1974): *Formal Philosophy: selected papers of R. Montague*, Yale University Press, New Haven.

Peregrin, J. (2000): 'Variables in Natural Language: Where Do They Come From?', M. Böttner & W.Thümmel (eds.): *Variable-free Semantics*, Secolo, Osnabrück, 46-65.

Peregrin, J. (2001): *Meaning and Structure*, Aldershot, Ashgate.

Peregrin, J. (2004): 'Pragmatism und Semantik', Fuhrmann, A. & Olsson E. J. (eds.): *Pragmatisch denken*, Ontos, Frankfurt a M., 2004, 89-108.

Peregrin, J. (2005): 'Is compositionality an empirical matter?', M. Werning, E. Machery & G. Schurz (eds.): *The Compositionality of Meaning and Content*, Ontos, Frankfurt, 135-150.

Peregrin, J. (2006a): 'Meaning as an inferential role', *Erkenntnis* 64, 1-36.

Peregrin, J. (2006b): 'Developing Sellars' semantic legacy: meaning as a role', P. Wolf and M. Lance (ed.): *The Self-Correcting Enterprise:Essays on Wilfrid Sellars*, Rodopi, Amsterdam, 257-274.

Peregrin, J. (2007): 'Consequence & Inference', V. Kolman (ed.): *Miscellanea Logica: Truth and Proof*, FF UK, Prague, 1-18 (`http://www.miscellanea-logica.info`).

Peregrin, J. (to appear): 'What is *the* logic of inference?', *Studia Logica*, forthcoming.

Pleitz, M., B. P. Göcke and H. von Wulfen (to appear): 'Possible Worlds in Terms of Incompatibility: An Alternative to Brandom's Analysis of Necessity', *The Logica Yearbook 2007*, Filosofia, Prague

Quine, W.V.O. (1960a): *Word and Object*, MIT Press, Cambridge (Mass.).

Quine, W.V.O. (1960b): 'Variables explained away', *Proceedings of the American Philosophical Society* 104, 343-347.

Quine, W.V.O. (1969): *Ontological Relativity and Other Essays*, Columbia University Press, New York.

Restall, G. (2000): *Introduction to Substructural Logics*, Routledge, London.

Wittgenstein, L. (1956): *Bemerkungen über die Grundlagen der Mathematik*, Blackwell, Oxford; English translation Remarks on the Foundations of Mathematics, Blackwell, Oxford, 1956.

Wittgenstein, L. (1953): *Philosophische Untersuchungen*, Blackwell, Oxford; English translation Philosophical Investigations, Blackwell, Oxford, 1953.