Logical rules and the a priori: good and bad questions

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Abstract. Many philosophers ponder the question of the a priori status of logical rules like modus ponens. In this paper I argue that the way in which such questions are usually posed is misguided. I argue that to accept modus ponens is nothing else than to have an implication; hence that to ask how do we know that implication obeys modus ponens does not make sense. To ask whether modus ponens is a priori is to ask whether having implication is a priori. I argue that as one is not born with a language (but perhaps only with a predisposition to acquire a certain kind of language, namely one possessing logical vocabulary and hence displaying logical structure), logical rules can be a priori only in the sense that we tend to acquire language governed by such rules.

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1 Boghossian’s question

In a paper examining the question of the a priori character of the knowledge of logic, Paul Boghossian ([1]) poses the following question:

To keep matters as simple as possible, let us restrict ourselves to propositional logic and let us suppose that we are working within a system in which modus ponens (MPP) is the only underived rule of inference. My question is this: is it so much as possible for us to be justified in supposing that MPP is a valid rule of inference, necessarily truth-preserving in all its applications?

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His hope is that if he can answer this question in the affirmative, and if he can answer also the follow-up question regarding the nature of the available justification (empirical or non-empirical?), then he will be clear about the a priori/a posteriori character of MPP, and perhaps, by way of generalization, about the a priori/a posteriori character of logic in general.

In this paper, we will follow the best traditions of analytic philosophy in that before focusing on the answer to

\[ (*) \text{ Are we justified in accepting MPP? (And is it a priori?) } \]

we will first scrutinize the meaning of this question. In particular, we will concentrate on the meaning of “MPP” (i.e. of “the rule modus (ponendo) ponens”) and on whether there is a meaning of this term which would render the question (*) both

(i) meaningful

and

(ii) non-trivial, i.e. not-self answering.

It is clear that if such a meaning is not available, grappling with (*) would make no sense. And what I want to show here is that this is indeed the case – that every available meaning of “MPP” renders (*) either meaningless, or trivial and hence not really interesting.

2 MPP as a matter of an artificial language

What could we understand under Boghossian’s “MPP”? As Boghossian speaks about “a system of propositional logic” (hereafter PL), what he could have in mind is, prima facie, something like the following:

(1) For every pair of sentences \( A \) and \( B \) of the language of PL it is correct (truth-preserving) to infer \( B \) from \( A \) and \( \langle A \rightarrow B \rangle \) (where \( \langle A \rightarrow B \rangle \) is a name of the sentence which arises out of the combination of \( A \) and \( B \) by means of the implication sign “\( \rightarrow \)”).

However, on second sight it seems pretty obvious that this interpretation of “MPP” would not fulfill criterion (ii), for on this interpretation, there would be no reason to consider a positive answer to (*). (1) is a stipulation which we may decide to accept or not to accept for various practical reasons, but how can a stipulation be justified? If we do not accept it, then “\( \rightarrow \)” will probably not be an implication, or not an implication of classical logic, but so what? (Note that rejecting (1) would not even preclude our way to classical logic – we could simply use another sign for the classical implication.) The language of PL, of which (1) is a standard part, is our artificial construct and so are the rules we have let govern its symbols, especially “\( \rightarrow \)”. Hence there is no reasonable sense in which we could be justified, let alone a priori, in letting “\( \rightarrow \)” be governed by
MPP in the sense of (1) – and (*) would be straightforwardly answered in the negative.

It might be objected that what is in question is not the bare possibility of rejecting (1), but the possibility of rejecting (1) without depriving \( \rightarrow \) of the status of implication. Does this help? I am afraid not – for what grants “\( \rightarrow \)” the status of implication? There appear to be basically two possibilities: either it is inferential rules by which it is governed, or the truth table which fixes its semantics. In the first case MPP would be one of the rules constitutive of its being implication, so to say that we cannot reject (1) without depriving \( \rightarrow \) of the status of implication would boil down to the truism that we cannot reject MPP without rejecting MPP. And the second case is only slightly less trivial: if “\( \rightarrow \)” is associated with the truth table, then we cannot reject (1) simply because it describes one line of the table (namely the one saying that if \( A \) is \( T \) and \( B \) is \( F \), then \( A \rightarrow B \) is bound to be \( F \)) – and the impossibility of canceling our stipulation without canceling our stipulation does not have anything to do with the a priori/a posteriori status of any of our beliefs.

A similarly unhelpful move would be to say that what is in question is the possibility of rejecting (1) without abandoning classical logic (with “\( \rightarrow \)” as the implication). Again, (*) interpreted in this way would be straightforwardly self-answering. It is simply (co-)constitutive of classical PL that “\( \rightarrow \)” behaves the way it does, in particular that it obeys MPP; hence rejecting MPP simply is abandoning classical logic. In short, if we consider Boghossian’s “MPP” as a matter of a formal language, then its acceptance/rejection is simply a matter of our decision (which, in some senses, might be more or less reasonable, or more or less useful, but surely not justified in a non-negotiable manner). Hence this could hardly be the right sense of “MPP” in Boghossian’s question.

All of this seems to indicate that if (*) is to be nontrivial, then “MPP” cannot be conceived of as a matter of a formal, artificial language. The rules of languages of this sort are our creatures, we are free to set them up as we like, and it makes little sense to ask whether we are justified (let alone a priori) in accepting this or that rule. Of course we can say that we are justified in the sense that by adopting some rules we reach something useful or interesting, but this pragmatic sense is clearly not the sense of “justification” relevant for (*).

3 MPP as a matter of natural language

This may make us try to interpret “MPP” in a wholly different way: not as a matter of a formal language, but rather as a matter of a natural one. So let us try:

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\text{(2) For every pair of sentences } A \text{ and } B \text{ of English it is correct (truth-preserving) to infer } B \text{ from } A \text{ and } "\text{If } A, \text{ then } B". \]

However, whether this is valid appears to be an empirical question concerning English – hence to believe it would be reasonable only if we had some empirical
knowledge about English; and hence it can hardly be a candidate for an *a priori* knowledge. Hence this also can hardly be the right sense of his “MPP”.

What about, then,

(3) For every pair of sentences $A$ and $B$ of English it is correct (truth-preserving) to infer the proposition expressed by $B$ from that expressed by $A$ and that expressed by $⌜I f A, \text{then} B\⌝$.

There are basically two ways of reading the phrases *the proposition expressed by ...* in this claim: the rigid and the nonrigid one. In the latter case *the proposition expressed by $A$* would be just any proposition which happens to be expressed by $A$ at the moment of the utterance, and consequently (3) would come out as equivalent with (2), so we are left with the rigid reading. On this reading, (3) talks about definite propositions: the ones which are now (in the moment when I am writing this, and hence presumably, but not necessarily, also when you read it) expressed by the English sentences which become the instances of $B$, $A$, and $⌜I f A, \text{then} B\⌝$. Hence if we assume that propositions are some kind of Platonist entities, MPP is, on this reading, no longer an empirical thesis.

However, is it, on this reading, true? I am afraid not. For take $A$ to be *Fido is hungry* and $B$ to be *He is nervous* – then it would seem that $B$ is simply, as such, neither true, nor false, even in cases in which both $A$ and $⌜I f A, \text{then} B\⌝$. Moreover, an English sentence of the form *If $A$, then $B$* is usually taken as true also in many cases when the truth of $A$ and the falsity of $B$ is improbable, but conceivable (*If it rains, the streets are wet*). Such apparent fallacies are usually ascribed to ‘logical imperfections’ of natural language, in particular to the fact that the English *if ... then ...* is not a true implication, but only something more or less close to it. Therefore, to avoid them and to make (3) valid, we would have to replace *if ... then ...* by its better, ‘regimented’ version. Perhaps “$\rightarrow$” governed by the rules of the classical PL, especially by MPP? But then, it would seem, we have gone the full circle and are back at (1) ...

This brings us to a general problem: how do we recognize which expression of a natural language is to be taken as the implication that should be taken into account when pondering the validity of MPP? As for English, we usually take for granted that it is *if ... then ...* – but what has made us think so? Definitely it was the way it functions, in particular the fact that in a typical case we may rely on *if $A$ then $B$* and $A$ entailing $B$. Hence it seems we need MPP to identify implication in the first place; and asking *does ‘the expression which obeys MPP (plus perhaps something else)’ obey MPP* does not look like a very sensible enterprise.

The moral seems to be that if we interpret Boghossian’s “MPP” as a matter of natural language, then his question will again come out as self-answering, in the negative. The validity of MPP thus interpreted is either an empirical, or a basically dubious matter\(^1\). As a consequence, neither can interpreting “MPP” in this way render (*) as meaningful and non-trivial.

\(^{1}\text{Cf. [8].}\)
4 Is there a third possibility?

It seems that the only way to render Boghossian’s question both meaningful and non-trivial would be to relate MPP to a language which is neither natural (in the sense of being an empirical entity), nor formal (in the sense of being the product of our definitions). Is there such a language? What about, for instance, a Fodorian language of thought\textsuperscript{2}? (This is, to be sure, a language which is supposed to be empirical in the sense that it is wired in empirical, human beings; but seen from the viewpoint of such an individual being, it might seem to be a suitable medium of the \textit{a priori}.)

(4) For every pair of sentences $A$ and $B$ of our language of thought it is correct (truth-preserving) to infer $B$ from $A$ and $⌜IfA, thenB⌝$.

I leave aside any possible reservations about the very concept of a language of thought the reader might have (I share them – I find the concept utterly problematic). The basic problem is again the identification of the sentence $⌜IfA, thenB⌝$ of our language of thought. How do we recognize it, do we see it before our mind’s eye, with a sign like “→” in its middle? How could we avoid the regress diagnosed in case of English and not to have to identify the sentence in terms of MPP in the first place?

Analogous problems emerge when we try to interpret “MPP” as an immediate matter of propositions of the Platonist breed, i.e. not mediated by sentences of any language. Again, bracket any possible reservations and suppose that there is a Platonist world, it contains propositions and for every two propositions $A$ and $B$ it contains a proposition, call it $⌜IfA, thenB⌝$, which obeys MPP. Suppose, moreover, in the spirit of medieval metaphysicians, that our ‘intellect’ has some kind of a direct access to this realm. How does it recognize the proposition $⌜IfA, thenB⌝$? Does this proposition contain a sign like “→”? As signs of this kind are our, conventional matters, this does not appear to make much sense. So is it the case that the owner of the ‘intellect’ in question simply, in some indescribable way, ‘noninferentially knows’ that this very proposition is $⌜IfA, thenB⌝$? But what does this knowledge amount to, in the first place? It seems that the only available senses in which we can say that a proposition is an \textit{implication} is a ‘syntactic’ one or a ‘semantic’ one. The former amounts to the proposition’s consisting of such an such parts, one of which is the implication operator; the latter, as it clearly cannot amount to the proposition \textit{denoting} something (which would obviously start an infinite regress), cannot but amount to the \textit{behavior} of the proposition (i.e. that it \textit{inter alia} obeys MPP).

This seems to indicate that the only way to make MPP non-trivial in the way required by Boghossian’s considerations would be to assume that there exists (within the Platonist heaven? within (transindividual?) realms of mind(s)) something which is \textit{essentially} implication, independently of whether it fulfills MPP. Only then does it make sense to take MPP as a principle which might be a reasonable candidate for an \textit{a priori} knowledge. I do not think this idea

\textsuperscript{2}See [4].
is any more meaningful than the idea of somebody who is ‘essentially bald’, independently of the actual density of his hair.

5 To accept MPP is to have implication

Given the difficulties with interpreting “MPP” so that it would render Boghossian’s considerations nontrivial, let us try to reassess the entire situation. Let us, namely, accept that MPP is always inevitably related to a language. Languages have rules, and what is called MPP is a certain rule within the context of certain other rules. In particular, MPP is a rule for inferring a statement from two other statements:

\[ A, B \vdash C. \]

However, surely not every rule of this form deserves to be called MPP; what we should require is that \( B \) is the implication of \( C \) from \( A \). But what makes a sign into an implication? Surely not its shape or design, but rather the way it functions; perhaps that it fulfills, aside of MPP, also the following “maximality condition”:\footnote{This appears to be a minimal requirement (cf. [7]); we may want to require something stronger.}

\[ \text{if } A, D \vdash C \text{ for a } D, \text{ then } B \vdash D. \]

Alternatively, we can say that a \( B \) is the implication of \( C \) from \( A \) if its truth conditions are in a certain way determined by those of \( A \) and \( C \) – in the simplest case that it is true just in case \( A \) is false or \( C \) is true. (We do not claim that the only reasonable implication is the material one; however, it would be hard to understand why something should be called “implication” if it did not fulfill at least the “only if” part of this condition – i.e. the condition that whenever \( A \) is true and \( C \) is false, it is false.)

In both cases, the concept of MPP comes to be inextricable from the concept of implication. Hence it seems that accepting MPP and acquiring an implication are simply two sides of the same coin. Viewed thus, the question whether MPP could be justified or whether it is a priori simply is the question whether we are justified in acquiring an implication, and whether this acquiring is ‘inevitable’. And it follows that the sense of the question is rather obscure.

Can we have a language without an implication? It depends on what we call “language”. There surely is a sense in which there can be a language without anything even remotely resembling implication. However, presumably every natural language (and every formal language which can be used to reason) has something close to implication. Hence if we agree that thought and language are two sides of the same coin, we can say that to be reasonable is to have a reasoning-apt language, especially a language with implication, and hence a language with MPP.
Hence do we accept MPP \textit{a priori}? Again, it depends on what we call “\textit{a priori}”. I do not believe that we are born with MPP in our heads – if not for other reasons, then simply because rules of the kind of MPP presuppose language and we are born with no language. But presumably we are born with a disposition to acquire a certain kind of language – which, besides others, contains implication (a word or a construction governed by MPP). Once we acquire the language, we come to accept MPP without having any external justification (for the most basic rules of our language are followed, as Wittgenstein ([14], §219), put it, \textit{blindly}) – so in this sense it is \textit{a priori}. But this kind of \textit{a priori} is wholly, to use Horwich’s ([6]) term, “semantogenetic”; and I do not think this is the sense of \textit{a priori} Boghossian and the majority of other authors recently addressing the topic would have in mind.

The moral we should draw is, I believe, that knowledge of logic is inseparable from knowledge of some language. Therefore if we do not believe in an inborn language of thought (a belief which I found contravened by Wittgenstein’s and Davidson’s analyses resulting into the claim that a language in the proper sense of the word is inseparable from the enterprise of intersubjective interpretation), we must conclude that the only sense in which knowledge of logic can be \textit{a priori} is that we are born with predispositions to acquire languages of a certain sort, and that we cannot become what we call reasonable without acquiring such a language. This does not mean that there is not a sense in which logic is universal: logic spells out a structure which must be exemplified by anything which is worth being called language and which underlies human reason.

6 The emergence of logic

An objection to this conclusion might be that explicating logical validity in these “semantogenetic” terms is bound to fall prey to Quine’s well-known criticism of (Carnapian) conventionalism ([11]). We cannot, Quine pointed out, assume that we know logical truths simply by knowing (and especially by having stipulated) the meanings of logical constants, for to get from the latter to the former we must apply logical rules which are one side of the coin the other side of which are logical truths.

This, though, is an objection against the possibility of creating logical truths out of the blue, by conventionally endowing certain signs with certain meanings. And the conclusion reached above should not be read as claiming the contrary: in no way do I mean that we could “semantogenetically” create logic out of a starting point devoid of any logic. The fact is that we, reasonable beings, do have a logic from the beginning (that is from the point when we are ‘reasonably taken as reasonable’). But, this logic is embodied within our language. We cannot conventionally turn artificial symbols into logical constants, by endowing them with appropriate meanings, without already having some symbols endowed with meanings which are capable of inducing a ‘logical space’, in which we can ‘move’ from a claim to its negation, we can draw consequences or quantify. (This is
the very space Sellars famously called the space of reasons\(^4\)) A language of this kind is passed over to us by our elders and is a necessary precondition of any explicit conventions we may want to accept later.

However, as Hellman already pointed out ([5]), the fact that we cannot draw the conclusions entailed by our stipulations without the help of logical rules does not contradict the fact that the stipulations have the conclusions. (Or if you prefer saying that they do not even have the conclusions before there is an explicit logic around, then we would have to say that there is no emerging of the logic without emerging of the conclusions – but to me this idiom appears to be rather mind-boggling.) Hence, we can say, we do not create logic by the stipulations, we only give it a guise in which it can make its appearance.

This is related to the famous rule-following discussion: we cannot follow an explicit rule without interpreting it, and to interpret it, we need further rules. As a consequence, not all the rules we follow can be explicit (and hence requiring interpretation). This means that before we can have explicit rules, we must have rules that are implicit to our practices. And similarly, before we can explicitly create logical vocabularies, we must be in an implicit possession of one.

This is, hence, the solution of the *prima facie* vicious circle. Though we cannot conventionally create a language without already having a language, we can develop (and we have done so) a language starting from a language-less state by coming to treat actions of others as right and wrong and consequently instituting (first rudimentary, and then more complex) linguistic rules. Parallelly, though we cannot do explicit logic without already being in possession of an implicit one, we can develop an implicit logic from nothing (i.e. become rational creatures) – simply by developing language with a logical structure.

### 7 What is logic?

Where do these considerations take us with respect to the question *What is logic?* It might seem that the ensuing conception of logic is an utterly empirical one: logic amounts to a structure which has happened to be shared by (most? all?) languages as they are currently actually used by human communities. Do I propose that doing logic is simply searching out linguistic universals?

No – this is only a *part* of the story. Logic is a matter of *rules*, and rules are Janus-faced entities (in fact understanding what a rule is simply is grasping, at least implicitly, its Janus-facedness). Rules are human creations and hence they are, in this sense, plainly an empirical, contingent matter. However, in contrast to other empirical entities, rules have an additional peculiar and unprecedented property: we can bind ourselves by their means, and to do so means to cease to take them as merely something contingent, and start to perceive them as something which determines how things *should* be or what we *should* do. Indeed, the very act of binding can be seen as a matter of (intentionally) giving up their contingent view – as, we may say, ‘dis-contingenting’ them.

\(^4\)See [13], §36; cf. also [2].
Hence, we can say, while viewed ‘from outside’, any system of rules is simply a contingent human formation, viewed ‘from inside’ it is a necessity (in the sense of commitment). Therefore, saying that

\[(**): \quad B \vdash A \rightarrow B\]

is typically not stating the contingent fact that a community happened to decide to use the sign → in a certain way (namely to adopt the rule that the formula \(A \rightarrow B\) is inferable from \(B\)); it is saying that it is correct to infer \(A \rightarrow B\) from \(A\). It is correct for me and you to the extent to which both of us consider ourselves members of the community of classical logicians accepting its rules. (If you do not consider yourself a member of this community, you are left with the empirical reading of (**), seeing it as a report about the members of some alien community.)

Hence studying logic is studying certain rules of our contingent language – which we, however, have ‘dis-contingented’ by that we have submitted to them. Logic, therefore, tells us how things should be. (It tells it only to those of us who play a linguistic game with the very rules; but as there is no way of being rational save playing a game with this kind of rules, we can leave out this restriction for it is simply implicit to the particle us.)

These considerations open up a vast cluster of problems centered around the concept of rule. Many other papers would have to be written to discuss all of them\(^5\). The only point I want to make here is that the emergence of rules makes room for a wholly new and unprecedented kind of idiom – not only the words such as “right” or “wrong”, but also the whole normative mode of speech. It was Kant who realized that rules are so exceptional that they may be used to draw the boundary between the mind-less object of the physical world and us, mind-full rational and responsible agents. He drew it as the boundary between the realm of law, where things blindly obey their kinds of rules, and the realm of freedom where we can follow our rules willfully. As Brandom put it:

> Descartes had bequeathed to his successors a concern for certainty: a matter of our grip on concepts and ideas – paradigmatically, whether we have a hold on them that is clear and distinct. Kant bequeaths to his successors a concern rather for necessity: a matter of the grip concepts have on us, the way they bind or oblige us. ‘Necessary’ [notwendig] for Kant just means “according to a rule”.

This notion of logic, i.e. logic as a matter of study of rules as ‘dis-contingented’ regularities, can be contraposed to the notion which Wittgenstein ([15], §I.8) ridiculed as the notion of logic as an “ultraphysics”:

> Die Logik ist eine Art von Ultra-Physik, die Beschreibung des ‘logischen Baus’ der Welt, den wir durch eine Art von Ultra-Erfahrung wahrnehmen (mit dem Verstande etwa).\(^6\)

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\(^5\)I discuss them in greater detail especially in [9] and [10].

\(^6\)“Logic is a kind of ultra-physics, the description of the ‘logical structure’ of the world, which we perceive through a kind of ultra-experience (with the understanding e.g.).”
(It is not difficult to find an exemplification of this standpoint in Wittgenstein’s vicinity – it was Russell’s ([12], 169-70) conviction that “logic is concerned with the real world just as truly as zoology, though with its more abstract and general features”.

This notion of logic stems from the fact that the laws of logic do not restrict us in the ways the laws of nature do. It is not that were my mind to try to eschew the constraints of logic, it would be prevented from so doing by a logical analogue of electric resistance or mechanic force. Rules of logic are restrictive because it is only within the ‘inner space’ they constitute that one can argue and indeed think (in the peculiar human way). Nothing can stop us from violating the rules of logic – but the price is high, namely quitting the community of rational agents, responsible for their actions.

8 Conclusion

This indicates that the question whether the rules of logic are a priori is ill-conceived: after what we learned from Wittgenstein, Sellars and others it is clear that a rule simply is not the kind of thing for which it would make sense to be a priori. Things which can be inborn are tendencies, inclinations, attitudes, ways of thinking and the like – but not rules. (We can be possibly born with propensities to learn a certain kind of language or to acquire a certain kind of beliefs – but not with the language or the beliefs themselves.) Rules can emerge and exist only in an intersubjective space, underpinned by a communal interaction.

This means that neither can we justify logical rule – justifying anything presupposes logic in the sense that we cannot justify without a language worth its name; and a language worth its name must embody logic. We cannot justify unless we already are in the space of reasons; we cannot be in the space of reasons unless we have a language; and nothing is a language unless it embodies the logical structure, i.e. unless its constitutive rules comprise rules constitutive of the space of reasons.

This is not to say that we cannot tell a story explaining how it has come to the situation in which we now do have logic. The story might begin with a tale (hinted at above) about how our ancestors could have come to be rule-followers and might continue with another tale about how we, by developing the rules of what we now call logic, were able to outsmart our competitors, and hence have made our logic into a success. (To put it anecdotically, those who did not employ an → so that A, A → B ⊢ B were weeded out by natural selection.) This is a kind of justification, though a thoroughly pragmatic one; and first of all it is not a justification of MPP, but rather of the claim that it is good to obey MPP (more precisely to possess a language including an implication and hence MPP).

This view also rehabilitates the view of logic as a study of certain rules, which was, during the history of the subject, often superseded by its view as the study of some very general structure of either the world or human mind
Logical rules of the kind of MPP are not our means of representing logic which itself resides elsewhere, but rather they are themselves the very, and the only possible, residence of logic.

To summarize: our analysis of the possible meanings of “MPP” has led us to the conclusion that asking (*) is asking a bad question, for it is asking a question which, in so far it makes sense, cannot but be trivial. The impression that it is not trivial is, I think, caused by the persistent picture of a priori truths being planted (by a god or natural selection) into our minds – a picture which nowadays almost every philosopher explicitly rejects, but which still sits on many philosophical noses like glasses so that everything the owners of the noses see, they see through them. I think that the only way our minds can be reasonably seen as prearranged is not by being endowed with particular contents, but rather by certain skills (or predispositions to acquire skills), especially particular linguistic skills. In this way we can say that to be reasonable is to develop these dispositions and acquire a ("true") language, i.e. a language with a logical structure, especially with something governed by MPP and hence being an implication.

References


