Introduction

The beginning of this century hailed a new paradigm in linguistics, the paradigm brought about by de Saussure's *Cours de Linguistique Générale* and subsequently elaborated by Jakobson, Hjelmslev and other linguists. It seemed that the linguistics of this century was destined to be structuralistic. However, half of the century later a brand new paradigm was introduced by Chomsky's *Syntactic Structures* followed by Montague's formalization of semantics. This new turn has brought linguistics surprisingly close to mathematics and logic, and has facilitated a direct practical exploitation of linguistic theory by computer science.

One of the claims of this paper is that the post-Saussurian structuralism, both in linguistics and in philosophy, is partly based on ideas quite alien to de Saussure. The main aim then is to explain the ideas driving the formalistic turn of linguistics and to investigate the problem of the extent to which they can be accommodated within the framework of the Saussurian paradigm. The main thesis advocated is that the point of using formalisms in linguistics is more methodological than substantial and that it can be well accommodated within the conceptual framework posited by de Saussure.

1 De Saussure vs. Structuralism

Before beginning to discuss structuralism, let us stress the distinction between the genuine views of Ferdinand de Saussure and the teachings of his various avowed followers, be they linguists or philosophers. In fact, de Saussure's theory, as presented in his *Course*, is an austere and utterly rational scientific theory articulated with a rigour commonly associated with linguistic theories of the 'post-Chomskian' period, though differing from them by the absence of
formalisms. Many of the de Saussure's followers tried to turn his approach into something quite different: into a tool of questioning scientific rationalism overemphasizing the "literary" aspect of language.

This is true particularly of French philosophers who used the structural insight to fight the analytic approach of their Anglo-American colleagues. It is beyond doubt that French structuralism constitutes one of the most significant philosophical movements of this century; however, its affiliation to de Saussure is an intricate matter. These philosophers have eagerly reassumed the view of language as a self-contained phenomenon to be explained by an appeal to its intrinsic properties; however, they have almost completely ignored other aspects of de Saussure's approach to language, notably his calm scientific rigour.1

Linguists such as Jakobson and Hjelmslev, of course, remained far more faithful to the teaching of their predecessor, but they failed to match his rigour. Thus Hjelmslev's theory, although guided by the promising goal of finding "the system beyond the process" and "the constancy beyond the variability",2 is overloaded with more or less mysterious concepts which he is not willing to make sufficiently precise; and Jakobson, although on the one hand ready for such exquisitely "Saussurian" claims as "if topology is defined as the study of those qualitative properties which are invariant under isomorphic transformations, this is exactly what we did in structural linguistics"3, on the other hand considers theory of language to be akin to literary criticism and claims that "only as a poetry is language essential"4.

2 De Saussure de-mythicized

In what sense then was de Saussure himself a structuralist? Structuralism, as developed by de Saussure, consists in viewing abstract linguistic objects (especially meanings, but everything that he calls linguistic reality) as values of elements of the system of the expressions that make up language. Let us explain this in detail5.

First, let us notice that to speak about a structure is possible only there where it is possible to speak about parts and wholes. Indeed: structure is the way of organizing parts into a whole. So to base one's theory of language on the concept of structure presupposes viewing language as a part-whole system.

Let us stress that the notion of a part-whole structure of language may be far from trivial. Expressions are indeed strings of words and as such they consist of substrings (thus John loves Mary consists of John loves and Mary, or of John and loves Mary, or of John and loves and Mary), but this trivial part-whole structuring is not what linguistics is about. Besides it there is another, nontrivial part-whole structure which can be imposed on the class of expressions of
language and which stems from centuries of investigations by grammarians. According to this notion John loves Mary consists of John and to love Mary, or of John, to love and Mary, where loves is considered to be only a kind of "form" (or a "manifestation") of to love.

Let us further notice that to speak about a structure is necessary only there where two different wholes may consist of the same parts. Indeed, structure then is what makes the difference. Otherwise there is no reason for not considering all wholes as having the same structure. We saw that the sentences John loves Mary and Mary loves John can be considered as consisting of the same parts. But these two sentences are different, and hence there must be something which makes them so; and it is this something that is addressed as their structure.

The part-whole view of language implies the perceiving of expressions as building-blocks, as constituents of more complex expressions, the ultimate wholes being sentences. (Sentences themselves can thus be viewed both as complete wholes and as blocks used to build more complex wholes.) Any block is suitable for some ways of building some wholes, and not suitable for other ways and other wholes; and the situation may arise in which the usability of two blocks coincides. This is the case when using one of the blocks instead of the other leads always to the result which we consider equivalent to the original one. (If we build houses and equate all houses of the same shape, i.e., differing only in colour, then we thereby equate also all bricks differing only in colour.) This is to say that considering some wholes equivalent engenders our also taking some blocks to have equal values.

Hence every equivalence on the class of expressions of language induces an assignment of values to expressions. The concept of equivalence, or, in de Saussure's term, identity, is thus interdependent with the concept of value. This is de Saussure's (1931, p.110) claim that "the notion of identity blends with that of a value and vice versa."

Now, roughly speaking, the main claim of de Saussure's is that all the abstract entities associated with expressions can be considered as values and hence as certain "spin-offs" (using the term as used by Quine) of certain equivalences (or oppositions, which are complements of equivalences).

3 Chomsky, Montague and Formal Semantics

Chomsky's path-breaking theory occasioned the reconstruction of language as a formal algebraic structure. Chomsky proposed to account for a language via a set
of formal generative rules, the recursive application of which to a given initial symbol generates all and only syntactically well-formed sentences of the language.

The notion of natural language as a bundle of rules is clearly nothing new. In fact, the very idea of grammar is based on this view: to write a grammar of a given language means to articulate rules accounting for well-formedness of that language. Chomsky's *novum* was that he proposed organizing the rules into a hierarchical system allowing for systematical generation, and basing all this upon setting up of the grammar as a real mathematical structure. Such a mathematization entailed an exceptional increase of rigour and perspicuity and, moreover, it led to the development of a metatheory, investigating into the formal properties of grammars (e.g. their relative strengths).

Chomsky's approach proved to be extremely fruitful in the realm of syntax, and linguists immediately tried to extend it to semantics. They attempted to generate meanings in the same way as Chomsky's theory generated surface structures. However, these attempts, be they presented as semantic markers of Katz and Postal (1964), or as generative semantics due to Lakoff (1971), in general failed to be satisfactory. The reason for this failure was diagnosed by Lewis (1972): it was the failure to account for truth conditions, which is a *conditio sine qua non* of semantics. Montague, Lewis and others therefore offered a new way to account formally for semantics based on the results of formal logic.

The basic idea was to treat meanings as set-theoretical objects on which expressions are mapped. The first approximation, going back to Gottlob Frege, was to reify the two truth values and to consider the meaning of a sentence to be directly its truth value. However, this approach had the unpleasant consequence that any and every pair of sentences that are either both true, or both false, are synonymous; which proves such an approach to be essentially untenable. The lesson to be learned seemed to be that the meaning of the sentence does not amount to its truth *value*, but rather to its truth *conditions*.

This obstacle was resolved by introducing the concept of possible world into semantics and this is where Montague enters the scene. (However, it is fair to stress that possible-world semantics was not discovered by Montague; he was neither the first one to use possible worlds as a tool of logical theory - the first to use them systematically were Stig Kanger and Saul Kripke - nor the only one to employ possible-worlds-based logic in an effort to formulate a systematic semantics of natural language; concurrently other theoreticians presented similar theories - at least Tichý's (1971) transparent intensional logic is surely worth mentioning. But Montague is the one who has become the legend.) Possible worlds were considered as the entities to which truth is relative; hence to say that the meaning of sentence was its truth conditions became to say that it was a certain function assigning truth values to possible worlds. This turned truth
conditions into entities accommodable within the framework of set theory.

The step from truth values to truth values relativized to possible worlds (and in general from extensions to intensions) was a good one, but not good enough. It soon became clear that even to consider every pair of sentences being true in the same possible worlds as synonymous is inadequate. Every truth of mathematics is true in every possible world; but it is surely inadequate to consider all truths of mathematics as synonymous. The solution accepted by the majority of semanticists was to consider meaning of sentence as something structured. According to an old proposal of Carnap (1957), two expressions were taken as really synonymous if they not only shared intensions, but were intensionally isomorphic, i.e. if they consisted of the same number of constituents and if their respective constituents shared intensions; and this idea, revived by Lewis, has served to ground the "hyperintensional" semantics that became prevalent in the eighties.

Lewis has proposed to consider meaning as a Chomskian tree whose leaves are occupied by intensions. His proposal has been further elaborated especially by Cresswell (1985). Other proposals to the effect of considering meaning as a structure, were articulated within the framework of situation semantics of Barwise and Perry (1983) and within that of discourse representation theory due to Kamp (1981). Tichý (1988) has reached the conclusion that the intension of a complex expression is constructed from the intensions of its components and has proposed to consider not the result of the construction, but rather the construction itself as meaning.

The shift from functions to structures hailed a rapprochement between the theories of logically-minded semanticists operating within set theory and those of the more traditionally-minded ones using formalisms more loosely. If we free ourselves from the 'ideologies' of individual schools, we can see that the gap between Kamp's discourse representation structure or Tichý's construction, on the one side, and Chomsky's deep structure or the tectogrammatical representation of Sgall et al. (1986), on the other, need not be crucial.

### 4 Language as an Algebra

Now what we are claiming is that formal linguistics does not in general bring insights essentially incompatible with the structuralist paradigm; rather, it carries out the "mathematization" of language thereby creating a framework in which the Saussurian point can appear quite intelligible.
We have stated that de Saussure's approach presupposes the view of language as a part-whole structure, i.e. as a class of items (expressions) some of them consisting of others. Thus we are viewing language as a class of expressions plus a collection of operations which enable more complex expressions to be made out of simpler ones. So, for example, the class of expressions contains the expressions John, to love, Mary and John loves Mary; and among the operations there is one that makes John loves Mary out of John, to love, Mary.

Now in order to state this in mathematical terms, let us present a tiny fragment of algebra. We shall restrict ourselves to three definitions, and we are not going to press for exactness; the aim of the whole enterprise is merely to illustrate the role which mathematics can play within a theory of language.

**Definition 1.** An algebra is an ordered pair \( A = <C,F> \), where \( C \) is a set, called the carrier of \( A \); and \( F = <F_j>_{j \in J} \) is a family of functions, called the operations of \( A \), each of which maps the Cartesian power of \( C \) on \( C \) (i.e. each of which is an \( n \)-ary function on \( C \)).

**Definition 2.** Let \( A = <C,<F_j>_{j \in J}> \) and \( A' = <C',<F'_j>_{j \in J}> \) be algebras with the same number of operations. Let \( G \) be a function from the carrier of \( A \) to that of \( A' \). We shall say that \( G \) is a homomorphism from \( A \) to \( A' \) if \( G(F_j(x_1,...,x_n)) = F'_j(G(x_1),...,G(x_n)) \) for every \( x_1,...,x_n \) from the domain of \( F_j \) and for every \( j \in J \).

**Definition 3.** Let \( A = <C,<F_j>_{j \in J}> \) be an algebra and \( E \) an equivalence (i.e. a transitive, symmetric and reflexive binary relation) on the carrier of \( C \). Let us call a subclass of \( C \) an \( E \)-subclass if each its two elements are equivalent according to \( E \). Let us call an \( E \)-subclass of \( C \) maximal if it is not included in another \( E \)-subclass of \( C \). Let \( C' \) be the class of all maximal \( E \)-subclasses of \( C \); and for every \( j \in J \) let \( F'_j \) be the function on \( C' \) such that \( F'_j(y_1,...,y_n) = y \) if and only if there exist elements \( x_1,...,x_n \) of \( C \) such that \( x_j \in y_j, \ldots, x_n \in y_n, x \in y \) and \( F_j(x_1,...,x_n) = x \). The algebra \( <C',<F'_j>_{j \in J}> \) is called the factor-algebra of \( A \) according to \( E \) and it is denoted by \( A/E \).

Using Definition 1 we can restate our previous consideration more mathematically. The consideration effectively views language as an algebra: the carrier of the algebra contains the expressions John, to love, Mary and John loves Mary; and among the operations of the algebra there is a ternary one, say \( F_k \), such that \( F_k(John, to love, Mary) = John loves Mary \).

Chomsky's contribution was not to abolish this view, but rather to
explicate it and to articulate the rules of the algebra of syntax in the rigorous
mathematical way. The contribution of the semanticists can then be seen in
articulating the semantic aspect of language into another, semantic algebra
connected with the first one by the function of meaning assignment.

The point is that if Chomsky's theory can be seen as the reconstruction of
language as the class of lexical items plus the class of grammatical rules
constructing more complex expressions out of simpler ones, then Montague's
contribution can be seen in mapping lexical items on certain basic set-theoretical
objects and in paralleling the syntactic rules operating on expressions by rules
operating on their set-theoretical meanings. Every expression $E$ is thus supposed
to be furnished with a set-theoretical meaning $\|E\|$; the simple expressions being
assigned their meanings directly, the complex ones via semantic rules. If we - for
the sake of simplicity - temporally shelve possible worlds, then we can say that
$\|John\|$ and $\|Mary\|$ are supposed to be elements of a basic set understood as the
universe of discourse, $\|to\ love\|$ is considered as a function assigning truth values
to pairs of elements of the universe (the sentence holds true for some pairs and is
false for other ones), and $\|John\ loves\ Mary\|$ as a truth value ($true$ or $false$).

The meanings of simple expressions such as $John$, $to\ love$, and $Mary$ are supposed
to be given in a direct way, whereas those of complex ones like $John\ loves\ Mary$
are considered to be "computable" out of meanings of its parts. Thus, the value
$\|John\ loves\ Mary\|$ is considered to be computable out of $\|John\|$, $\|to\ love\|$ and
$\|Mary\|$; namely as $\|to\ love\|\left(\|John\|,\|Mary\|\right)$. In general, the meaning of a
complex expression is a function of meanings of its parts. This approach thus
sanctions the so called principle of compositionality, which has been considered
basic for the theory of meaning since Frege.

The set-theoretical meanings of lexical items plus the rules to compute the
meanings of complex expressions out of their parts thus yield an algebra with the
same number of operations as the algebra of expressions. Meaning assignment
then comes out as a mapping $M$ of the algebra of expressions on the algebra of
meanings such that to every operation $F$ of the algebra of expressions there
Corresponds an operation $F'$ of the algebra of meanings such that for every $e_1,...,e_n$
from the domain of $F$ it holds that $M(F(e_1,...,e_n))=F'(M(e_1),...,M(e_n))$. Hence,
referring to the Definition 2, we can say that the meaning assignment is a
homomorphism from the algebra of expressions to the algebra of meanings.

---

STRUCTURAL LINGUISTICS AND FORMAL SEMANTICS   91
5 Saussure Mathematized

With the help of this framework, we find that many points previously difficult to articulate, become surprisingly simple. An example is the way we have just expressed the principle of compositionality: this principle, which has been constantly subject to misunderstandings, now becomes the simple and unequivocal claim of the homomorphic character of meaning-assignment. Everybody who is familiar with the basics of algebra easily understands; a misunderstanding is hardly possible.

We have stressed that de Saussure's claim is that the meaning of an expression is its value resulting from oppositions present in the system of language. We have stressed also that the value is a reification of the way the expression functions as a building-block for building wholes suitable for various purposes, notably true sentences. Translated into our algebraic framework, the algebra of semantics owes its being to certain oppositions present within the system of language, notably to the opposition between truth and falsity, or, which is the same, the equivalence of sameness of truth value.

Algebraic theory allows us to clarify how an algebra plus an equivalence between elements of its carrier yields a new algebra: our Definition 3 articulates this in explicating the term factor algebra; it amounts to the "coalescing" of the equivalent elements of the original algebra and to the corresponding adjustment of its operations. This suggests the idea of considering the algebra of meanings as the factor algebra of the algebra of expressions factored according to the equivalence of sameness of truth value.

The obvious objection to embracing this conclusion is that it leads to identifying meanings with classes of expressions, which seems to be highly implausible. However, saying that the algebra of meanings can be considered as an algebra of classes of expressions is not to say that meaning be a class of expressions - the point of the structural view is that meaning is not this or that kind of thing, that what there is to meaning is rather the structure of the algebra of meaning. This is to say, in algebraic terms, that the algebra of meanings is definite only up to isomorphism; the factor algebra of the algebra of expressions must be seen as a mere representative of the whole class of isomorphic algebras, each of which can be considered to represent the algebra of meaning, and none of which can be directly identified with it.

In fact, formal semantics can be seen as placing additional, pragmatic requirements on the algebra which is to be considered as the algebra of meanings; it endeavours to select that of the isomorphic algebras which would be the easiest to work with. In particular, it is usual to require that the operations of the algebra of semantics be as simple as possible. Frege proposed that the meaning of a
sentence should be considered as the result of application of the meaning of its predicate to those of its terms. This idea was subsequently generalized to yield the general requirement that the operators of the algebra of meaning should be operators taking as one of its arguments a function and yielding what this function yields when it is applied to the remaining arguments. This means that if $F$ is an $n$-ary operator of the algebra of expressions, then there exists an $i$ such that for every $n$-tuple $e_1, ..., e_n$ of expressions from the domain of $F$ it holds that

$$F(e_1, ..., e_n) = e_i(e_1, ..., e_{i-1}, e_{i+1}, ..., e_n).$$

The relation of an algebra supplemented by an equivalence to the corresponding factor algebra is thus the prototype of the relationship between the system of language expressions and a system of values which the expressions acquire with respect to some opposition or equivalence. Although the algebraic model, if taken literally, might lead to an oversimplified view of language, it evidently dramatically improves the intelligibility and comprehensibility of the Saussurian point of the value-like character of meanings and other linguistic abstracta. And, this is, in general, the true role of mathematics within a theory of language: it is neither to improve language, nor to give its precise and exhaustive description, but rather to facilitate comprehensibility and intelligibility of language via confronting it with models.13

6 Structuralism Rejoined?

De Saussure's structuralistic insights invited generalization: the idea of structuralism is far more sweeping than to restrict itself to linguistic reality. The French structuralists took one direction of generalization: they sensed that natural sciences were threatening to swallow up humanities and they have ended up with philosophy as a kind of literary genre (viz Derrida). But there were other people more fascinated by, than fearful of, the sciences and they instead merged their own structuralist insight with the rigorous scientific thinking. Analytic philosophers, from Russell, Carnap and Wittgenstein to Quine and Davidson, were, of course, not influenced directly by de Saussure, but their teachings seem to be in certain aspects more congenial to the spirit of *Cours de linguistique générale* than the teaching of those who are usually considered as de Saussure's direct followers. And it was analytic philosophy whose advancement is inseparably interlocked with the advancement of formal semantics.
I do not want to claim that analytic philosophy and formal semantics are necessarily intrinsically structuralist. The views of many analytic philosophers, and of even more formal semanticians, amount to a kind of nomenclatural view of language, which is the direct opposite of structuralism. Many analytic philosophers did not manage to resist the temptation to embrace a form of naive scientism and ended up with a kind of systematic metaphysics (now approached via language and done with the help of mathematics and set theory) which the structuralist insight vanquishes. But a significant number of these thinkers, people such as Wittgenstein, Quine or Davidson, avoided such traps and their approach can be truly called structuralistic.

Quine's (1992) recent conclusions about the concept of structure are even more radical than those of his predecessors: instead of urging the reduction of abstract objects to relations, he questions the very idea of an object: "The very notion of object, or of one and many, is indeed as parochially human as the parts of speech; to ask what reality is really like, however, apart from human categories, is self-stultifying. It is like asking how long the Nile really is, apart from parochial matters of miles and meters." (p.9) Hence aphoristically: "Save the structure and you save all." (p.8)

The views close to Quine's and especially relevant for what we have been pursuing here are due to Donald Davidson - it is him who has made it clear that formal semantics need not be understood as a naively-metaphysical, nomenclatural matter and who has shown the plausibility of the theory of meaning as derivative to the theory of truth. And it is these views which we try to show to be congenial to the basic insights of de Saussure.

We have stated that the ways of analytic philosophy and structuralism essentially parted. However, their recent offspring - post-analytic philosophy and poststructuralism - are no longer antagonistic and indeed are sometimes surprisingly close. I think that the inseparability of the question about the nature of reality from the question about the nature of the language we use to cope with the reality, as urged in the above quotation by Quine, is in fact the same problem as that which irritates people like Foucault and Derrida. And I think, on the other hand, that the "careful mathematization of language" which is urged by Derrida (1972) is nothing else than the non-nomenclatural, non-metaphysically founded usage of formal logic as pursued by Quine and Davidson.
7 Conclusion

De Saussure's structuralistic view of language is quite compatible with the formal trend of linguistics appearing during the recent decades. In fact formalization and mathematization help to make the structuralist point intelligible.

Many theoreticians believe that formal semantics and analytic philosophy is connected with a nomenclatural view of language and hence is incompatible with the structural insight. But this is wrong - formal semantics is in itself neutral, and it is capable of being explicated both in the naive nomenclatural way and in the way congenial to de Saussure's structuralism; and among analytic philosophers we can find outstanding representatives not only of the former, but also of the latter view.

NOTES

1 There are philosophers who evaluate even more harshly the way in which French structuralists handled the heritage of de Saussure. Thus Pavel (1989, p. vii) characterizes their efforts as follows: "They mistook the results of a specialized science for a collection of speculative generalities. They believed that breathtaking metaphysical pronouncements could be inferred from simple-minded descriptive statements."

2 Hjelmslev (1943, p.11).

3 Jakobson (1971, pp. 223-4).


5 For a more detailed expositions of the issues presented in this section see Peregrin (1994b).

6 Chomsky himself, of course, would consider his approach not a mere improvement of methodology, but as an empirical discovery concerning human's innate inner workings; we leave this conviction of his aside, because it is peculiar to his own line of thought and it is not essential to the formalistic turn as such.

7 Lewis claimed that linguistic theories of meaning are mere translations of natural language into another, formal language, namely 'markerese'. However, I think that this caveat, as it stands, is misguided; every explicit semantic theory is clearly a translation of natural language into another language, be it 'markerese', the language of set theory, or whatever. The only way to do explicit semantics is to make statements 's' means m, where m is an expression of a language.

8 For general information about the concept of possible world see Partee (1989); for
conceptual analysis see Peregrin (1993a).

9Kamp's framework aims, besides this, at capturing what can be called dynamics of language, especially its anaphoric capacities; and it slowly becomes a paradigm of the semantic theory for the nineties.

10This point was made quite clear by Davidson (1984, p.30): "Philosophers of a logical bent have tended to start where the theory was and work out towards the complications of natural language. Contemporary linguists, with an aim that cannot easily be seen to be different, start with the ordinary and work toward a general theory. If either party is successful, there must be a meeting."

11Taking the intensional aspect of language at face value, we have to relativize all of this to possible worlds: the denotations of \[\text{John}\] and \[\text{Mary}\] (if we do not treat them as rigid, i.e. possible-worlds-independent, designators) will be functions from possible worlds to the universe, \[\text{to love}\] a function from possible worlds to pairs of elements of the universe, and \[\text{John loves Mary}\] will be a function from possible worlds to truth values: in some worlds (situations, time-spans etc. the sentence holds true, in other worlds it does not.

12The objection that such an explication is simple only due to the backlog of the complicated theory of algebra, is not sound - algebra is nothing \textit{ad hoc}, it is a well established theory whose meaningfulness is independent of whether we do or do not use it within a theory of language.

13See Peregrin (1993b).

14For Davidson's way of understanding semantic theory see Davidson (1984); see also Peregrin (1994a).

15The recent philosophical development of Richard Rorty documents that these two seemingly disparate approaches to philosophy could lead to a unified stance.

REFERENCES

Hjelmslev, L. (1943): Omkring sprogteoriens grundlaeggelse, Copenhagen.